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Can Frequent Use Of Number Talks Increase The Comprehension, Understanding, And Fluency Of Fractions, Decimals, And Percentages In Alternative High School Students?

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CAN FREQUENT USE OF NUMBER TALKS INCREASE THE COMPREHENSION, UNDERSTANDING, AND FLUENCY OF FRACTIONS, DECIMALS, AND PERCENTAGES IN ALTERNATIVE HIGH SCHOOL STUDENTS?

By

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DEDICATION

I dedicate this to my students, past, present, and future. Education is a lifelong endeavor; I strive to be an example to all of you and be worthy of your respect.

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ACKNOWLEDGMENTS

First and foremost I would like to acknowledge my wife, Jessica. She is the one who makes me better at everything I do. She is my first editor, my companion, and my best friend. She continues to try to educate me on how to use semi-colons as well; after 15+ years of trying to do that you would think I would have learned by now. She was with me when I first started graduate school in 1999 and she is with me now as I finally finish my Master's Degree. I promise, no more big papers for a while.

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CHAPTER ONE

Introduction

Math can be a difficult subject for many students. Scores and grades from years of teaching indicate that math, for many of our students, is unapproachable, boring, and confusing. Over the last two years, through professional development, I was introduced to the concept of math/number talks. (For the sake of convenience in this capstone, I will use the terms number talks and math talks interchangeably. There is relatively little difference between the two terms; both are used to discuss math concepts and both are used to increase student engagement.) How can number talks affect the learning of our students, especially those most in need of help? I believe that actively engaging students in discussions about math will help even the most reluctant learners in the classroom, those who historically have failed in math classes.

Math teachers face a lot of issues with their curriculum. It can be difficult to impart the importance of mathematics to most kids and we are often faced with an unending stream of questions revolving around the purpose of math for our students. Couple this with parents who had similar experiences with math, and we have a recipe for a concerted effort to thwart the importance of this vital subject to our students.

"Number talks" give math teachers another tool to use to help engage their students and get them talking about math instead of just "doing" math (Parrish, 2012; Humphreys & Parker, 2015). It is vital that students see a need for learning this discipline and without dynamic, engaging lessons, math will continue to be unapproachable, boring and confusing (Boaler & Dweck, 2012).

Who I am

My story about math is not unique among many in our system. From very early on math was something I could do with aplomb. I had my struggles, but I always managed to get good grades and pass my classes. Math was something I did, not something I loved or enjoyed. I might even say I never really even understood it. Some of my earliest memories revolved around concepts that would inform on how I think about math to this day. I had a few lessons here and there that sparked interest but they were few and far between. I still remember tearing the corners off a triangle and putting them together to form a half-circle. This simple activity had a profound affect on my understanding of triangles. These events were few in my math education, though.

I remember tediously sweating through numerous proofs in geometry in tenth grade and celebrating when the teacher told us we did not have to keep proving certain theories in the quest to finish a proof; for me it was yay, one less step. I, like many geometry students, chafed at the idea of having to re-prove well-established theories and axioms. Unlike many of my peers, however, I never struggled with the proofs in geometry, largely because I could retain much of the information about what we learned easily. I "got" proofs, but never really understood them.

As I graduated high school I found myself drawn to the sciences and Purdue University. I was about to start into a highly technical and rigorous major in Chemistry. I did not test well enough to start calculus in my first semester so I found myself retaking pre-calculus. I spent the first half of the semester rehashing previously learned information and getting bored with math again. I should have taken the opportunity to fully understand the material because I would have been better prepared for calculus at Purdue.

Calculus at Purdue is hard; there is no other way to state it. I struggled and struggled and struggled. I never got it. I failed at math in a big way. This was unnerving for me because math was always something I got; the fact that I did not in this instance really disturbed me. After struggling with math I found a path to a degree that did not require calculus. To this day I state the reason I am not a chemist is calculus; it had become my bogeyman.

My return to math came many years later with maturity. I found my path to a profession I liked, history, and a vocation I wanted to follow, teaching. Like many, I had trouble finding permanent employment as a social studies teacher. Lightheartedly, I liked to say that you could not throw a rock in Minnesota without hitting five-out-of work social studies teachers. However, I found myself subbing at the local district, particularly in math. More importantly I was being noticed for teaching it well and having a good grasp of the concepts. I was finding I was good at math again.

As I gained this newfound confidence I was approached by a colleague who simply said, "You're pretty good at this, why don't you get a math license and teach math?" A question, seriously asked, with a host of baggage on my part; could I even contemplate a return to the one subject I just could not get?

After some reflection and some discussions with my wife I decided to look into what it would take to get a math license. I would need to complete a math major, take a math teaching class, and be a student teacher again. A math major, that means calculus…. The bogeyman had returned. I started small, hoping to get a solid foundation from which I could battle my foe. I took a college algebra class and aced it. I took a trigonometry class and aced it. I was finally face-to-face with my old nemesis, calculus; I would need to be successful this time around if I wanted my plans to come to fruition.

With trepidation I approached my old foe, and found insight, amusement, and appreciation for the beauty of a discipline that continues to amaze me. Something clicked; calculus made sense and math was my friend again. Not only was my understanding helpful in doing well in my classes, it was transformative. I saw a side of math I never saw before, the clear unyielding clarity that comes with understanding the whole picture. I now saw beauty and purpose in a place where before I had only seen tedium and repetition. Two years later, and many math classes, I had a new license and a greater understanding of math.

I found another bonus to this new branch of my career; I was hired as a math teacher two weeks after getting my license. I would have my own classroom, and I would be a math teacher. I now had the chance to show others the beauty I now saw and understood. Not only did I get calculus; I wanted to teach it.

Why I am doing this

After a year at a local high school I was hired at an alternative high school and this is where my math journey truly flourished. Here I met kids who had struggled with math for years, had gaps in their knowledge and were not armed with the proper tools to ensure success in math. My first year, I taught probability and statistics and geometry at the alternative school. I was amazed at the lack of understanding of basic fundamentals and how that translated into poor performance and many misunderstandings about math. This problem was particularly evident in statistics and probability. The minute the students saw fractions on the board they were done. Never learning fractions to the point that they were comfortable meant monumental struggles in calculating probabilities.

For several years I also struggled to help these students with probability, ever searching for a means to help them acquire the fluency with fractions that they needed. I attended numerous professional developments meant to help students increase their abilities and pass math. In all honesty, math instruction has changed very little since I was in high school and I had classrooms full of students who proved that the current system was not reaching as many as we hoped. Two years ago our district made a push to bring more discussion and student engagement into math. This push was something called a math or number talk. It was a fairly new approach that encouraged student thinking and problem solving. As primarily a stats teacher, I did not immediately see the effectiveness for this in my classroom. Most of the other district teachers saw the uses in forming curriculum-driven discussions about what they are teaching, but I did not see a connection to how this would translate for my class. After coming up with a few cursory activities that could be connected to stats I went home with no real intention of ever using these techniques.

As with many new ideas and initiatives, the true use was not apparent at first. As I thought about my students and their abilities it began to dawn on me that maybe using these talks for driving the curriculum was not the most effective use of these strategies. What if I used these talks to shore up vital abilities in mathematics that my students may not completely grasp? With that thought in mind I attended another PD devoted to math talks and started coming up with mini-lessons that could be taught at any time throughout the trimester to my stats students. These lessons would include discussions on fractions, decimals and percentages, as well as basic number theory to help develop abilities that may never have formed among my students.

As with any new technique I began to experiment with my classes. At first I tried some simple tasks involving multiplication. I found that these fostered more discussion if I told the students that calculators were not allowed. I wanted them thinking and doing, not punching numbers on a device. At first the students were hesitant, not really sure what I wanted. So I started modeling discussions with them and they started to catch on to what I wanted. The students began to participate. I started seeing authentic discussions and intriguing strategies to solve the day's questions. I even started seeing ways to integrate the idea into my curriculum. Here and there discussion started to find its way into my classrooms.

The value really came together for me after I was observed during one of my performance cycles by one of my administrators. She observed on a day that I fostered discussion on the Fundamental Counting Principle. This was the first time I used a math talk strategy to help the students figure out how the rule worked. During the observation debrief she was amazed at the discussions the students had and their commitment to sharing ideas. At that point I was sold on the effectiveness of math talks to increase student participation and understanding in math.

Getting students to discuss math and how numbers work in their own life makes the concept applicable and thus, relevant. Relevance is key to at-risk youth; these students need a concrete reason to learn the topic. One topic we have applied is number sense, specifically how to calculate a clearance price or a tip at the restaurant. The ability to fluidly change from fractions to decimals and percentages is a skill that can be used throughout life. My personal observation is that many of these kids do not understand this fluidity. To quote a famous mathematician, Edward Frenkel (who got it from one of his teachers, Israel Gelfand), "If you ask a drunkard what number is larger, 2/3 or 3/5, he won't be able to tell you. But if you rephrase the question: What is better, two bottles of vodka for three people, or three bottles of vodka for five people? He will tell you right away: Two bottles for three people, of course" (Frenkel, 2013). If you forgive the inappropriateness of the example, this is a powerful concept to apply to many aspects of life.

How I am doing this

The genesis of the idea for using this as a capstone topic came from my action research project. *Could frequent use of number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?* This question intrigued me and guided me to research on math talks, student discussions, and strategies for reaching students in an alternative setting. Now that I had an idea I needed a strategy to implement my research.

I know that I need to identify what "frequent" means in my question; I figure twice a week is the right amount. Next, I believe that a pre- and post-test will allow me to determine if a student is making progress towards understanding fractions, decimals and, percentages. In addition to the tests I will also ask students to reflect on their learning to determine if the talks have any effectiveness. Additionally, it makes sense to have another teacher observe the number talks and look for certain student behaviors. Using the dual approach of quantitative and qualitative data I can more readily determine the effects of these discussions (Creswell, 2014). My research methodologies are covered in greater detail in Chapter 3.

In Chapter 2, I detail the literature I studied for researching the question; *can frequent use of number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?* In the literature review, I outline the strategies used to engage at-risk youth as well as the effect of math/number talks with elementary and middle school students. There is also a discussion about how we teach and how students learn about fractions, decimals and percentages. Lastly, I cover the effect of discussions on the achievement of alternative high school students and tie all of this to the research I lay out in Chapter 3.

CHAPTER TWO

Literature Review

In reviewing the literature for this capstone, it seemed best to approach this topic from multiple angles. The answer to *"Could frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?"* would not be easily answered, particularly for the population chosen. There are numerous examples of how this topic applies to elementary and middle school populations but there are very few studies that relate to high school or alternative schools. As a result, this chapter is split into four sections detailing the indirect way this question was researched given the lack of resources relating to the population.

In the first section, a review of the current literature relating to number talks is discussed, starting off with the seminal work on number talks and a specific strategy guide for doing number talks with fractions, decimals and percentages from Parrish (2010, 2016), and that of Humphreys and Parker (2015). Although they were not the first to do number talks they were among the first to provide detailed descriptions of what they should be and how to include them in a classroom. This section will include two subsections: how to conduct number talks and what their effects are on classrooms.

The second section describes why fractions, decimals and percentages can be difficult for students to learn. The section will describe the book by Susan Lamon, *Teaching Fractions and Ratios for Understanding* (2008), and other authors detailing how fractions are taught and, more importantly, how students learn about and use fractions. This section will also detail student thinking about fractions and how misconceptions can lead to problems applying that knowledge after they first learn it in elementary school. Additionally, there will be a discussion on proportional thinking in regards to fractions, decimals and percentages. Many students attending these schools have incomplete knowledge of mathematical concepts and this section will strive to explain how that disconnect occurs and what educators can do to help correct these misconceptions.

The third section details a definition of what it means for a school to be considered alternative and how these schools approach pedagogy and curriculum design. It should be noted that alternative schools are often at the forefront of curriculum changes, largely because the populations they deal with require teachers to be innovative in implementing and designing curriculum. Not every student "fits" in a traditional school and it is a benefit to all that these students have options that allow them to be successful in the alternative environment.

The last section details engagement, and specifically discussion, as a means to encourage student success. In this section a discussion on mathematical mindsets by Boaler and Dweck (2016) shows how changing student thinking in regards to math can be transformative. The ability to give students the freedom to make mistakes in a safe environment encourages brain growth and innovation in solving mathematical concepts. There will also be an examination of Hattie's (2012) research which details the massive effect discussion and engagement have on student success in math.

Number Talks

Definition and procedure. This section is in two parts; in the first is an examination of the importance and significance of the concept of what a number talk is and does. After the definition is established, an examination of the effect that number talks have on specific sets of students will be presented, the idea being that this will inform on what effect number talks will have on the experimental population.

What is the definition of a number talk? The idea seems to be pretty self-explanatory but the concept is much deeper and more profound than just talking about numbers. Too often in mathematics, the focus lies on getting a correct answer. Number talks allow teachers and students to explore how we get to that answer, and often, that the journey is more important than the destination. Number talks allow students to explore strategies in a risk-free setting, to discuss and reason their processes and compare one's own strategies with those of another, as opposed to merely completing the task. Higher order thinking about the process often gets left behind in favor of doing the process to get the answer (Humphreys & Parker, 2015).

In *Making Number Talks Matter*, Humphreys and Parker (2015) lay out a process, easily followed, that details what a number talk is and is not. The purpose of number talks, according to Humphreys and Parker, is to delve into student thinking and reasoning. Too often students blindly follow an algorithm without understanding why it works. Often students are taught to just use the algorithm in the hope that understanding will come with use. Number talks can enhance this understanding by helping explore the reasons why the algorithms work and expanding on multiple ways to find a way to the answer (Humphreys & Parker, 2015).

The procedure for number talks can be varied according to how the teacher wants to pursue the question, but Humphreys and Parker lay out a general procedure. First, the students put all materials away; this can and should be a mental activity. The teacher writes the problem on the board or document camera or whatever is available that allows students to see the question. Next, the teacher allows time for the students to work the problem using various methods to judge when the students are ready to discuss. This presents a nice option for differentiation as students who figure it out quickly can be asked to solve it in another way. When most thumbs are up the teacher then solicits answers, and only the answers, which are then written where everyone can see them. At this point students can defend the answers and how they reached them to the rest of the class, while the teacher merely acts as a recorder and moderator of the various methods; this aspect of the process is student-led and the teacher needs to allow that. Asking leading and higher order questions, the teacher pushes the students to rationalize their thinking and reasoning. When the teacher feels the questions' possibilities are exhausted they can return to the day's lesson. Ideally, these should be about 15 minutes but can go longer if the teacher wants (Humphreys & Parker, 2015).

This strategy turns the typical process of a math class over. Instead of being told how to do a problem, the students explain it to others. Instead of the teacher stating the strategy and having the students follow it, the students determine the strategy and share it with the class for discussion. As Humphreys and Parker state it, "Number talks are about students making sense of their own mathematical ideas" (p. 13). With this strategy, math teachers need to break the tradition of "doing the thinking for their students" (p. 14) Teachers need to use problems that allow students to use their current mathematical skills and put them to use in new ways (Humphreys & Parker, 2015).

Humphreys and Parker lay out some basic guiding principles for number talks. Teachers need to listen to all students because all students have valid insights. The purpose is to understand student thinking and to give students the freedom to make mistakes. Speed is good, but the thinking and learning of each individual learner is more important. Students need to feel safe sharing their ideas, which allows them to build social and mathematical skills. Understanding takes time to develop so allowing the students their confusion and the opportunity to struggle leads to learning. Lastly, you need to encourage creativity in solving the problems. It is important that students know there is no one “right” way to solve a problem. All ideas are valid if they work and the reasoning is sound (Humphreys & Parker, 2015). These strategies all play into the discussion and engagement strategies, which are discussed later in this chapter.

Humphreys and Parker echo many of the same thoughts as Parrish on how to conduct a number talk. Parrish also encourages classroom discussions and a supportive community in which to work these problems. Parrish emphasizes higher order questioning from the teacher and letting the students control the discussion in addition to letting the students use mental math and reasoning (Parrish, 2011).

Parrish also has several how-to books (2011, 2016) in the same vein as Humphreys and Parker. Parrish’s books also lay out the rules for how to conduct number talks in the classroom. Parrish delves into the procedure as well as strategies for engaging the students and encouraging good questioning in order to assist the students in making number talks meaningful. One of Parrish’s key components for a number talk as a teacher is to create a cohesive classroom community that is safe and risk-free for the students. Without this community, students would not feel comfortable trying new ideas and strategies. Alongside these new strategies the students must feel comfortable discussing these strategies with each other. These discussions give the students the opportunity to clarify their thinking, consider and test other strategies, investigate and apply mathematical relationships, build a repertoire of strategies, and make decisions about choosing strategies in specific situations (Parrish, 2010, 2016).

The teacher's role in this case, like with Humphreys and Parker, is to facilitate these discussions. Parrish states that the teacher assumes the role of, "facilitator, questioner, listener and learner" (p. 12). The teacher should keep the discussion focused on strategies and reasoning. The students lead this discussion, the teacher probes and pushes for understanding and comprehension (Parrish, 2010).

Parrish also stresses the importance of mental math in working with number talks. This is vital to helping students develop understanding of number relationships to solve problems, instead of relying on algorithms. Granted, algorithms are useful for solving problems, but often, these algorithms lack the nuance of understanding place value and number sense. A student blindly following an algorithm learns nothing about the patterns of how and why the systems work as they do. This fosters a reliance on an artificial means of solving a problem without really understanding why it works. Algorithms are useful for solving things but they do not help a student understand these relationships (Parrish, 2010).

Next, it is important to use "purposeful computation problems" (p. 14). The point behind this is to create problems that guide students to focus on mathematical relationships. Using problems that allow students to "break" them into easier problems improves understanding of how the systems work and relate to each other. For example, 19×4 can easily be solved using the standard algorithm; however, as a number talk students would be encouraged to approach this from various different angles by "breaking" numbers into smaller (for example 10×4 plus 9×4 or 20×4-1×4), more easily computed, numbers; this allows the students to solve the problem more quickly and easily than using the standard algorithm (Parrish, 2010).[1]

Parrish also lays out a specific procedure to follow for number talks. The teacher must choose a designated location for the number talk. This location should allow the teacher to maintain proximity to the students to allow observation and interaction with the students. One of the hardest tasks for many teachers is to allow appropriate wait time for your students to work the problem. Ideally, all of them should work it, but depending on the dynamics of the class some may finish sooner than others. The teacher must accept, respect, and consider all answers. This is where the discussion comes in; the students should explain their strategy and thinking behind the solution. At this point the teacher should allow the students to discuss what is going on and determine whether the strategy is sound, feeling free to provide prompts to the students to encourage discussion (Parrish, 2010).

In *Building Powerful Numeracy*, Harris (2011) points out the strategies for building math skills amongst middle and high school students. Harris states the different skills needed to build numeracy among students: flexibility, accuracy, creativity, algorithms, speed, and mental skills. Although three of these skills, accuracy, algorithms and speed, go against the ideas of number talks, the others play well with the philosophy of, and the ideas behind, number talks. In particular, flexibility, creativity and mental skills seem to be encouraged in number talks. Harris includes numerous strategies for developing and promoting numeracy in the book. These strategies use differentiated

[1] An algorithm is a "procedure for solving a mathematical problem in a finite number of

tasks meant to instill number sense into students. These tasks reflect the ideas presented by Humphreys and Parker. Mainly, having students try problems and then discuss the methods by which they solved them.

Another book, *Getting From Arithmetic to Algebra* (Schwartz & Kenney, 2012), gives some strategies for getting students from a strictly procedural understanding of math to a more complex conceptual understanding of number sense and number theory. Schwartz and Kenney (2012) discuss the point of mathematics and its education through elementary school. Understanding and processes are very important to early math education but as the students move to more complex topics surrounding mathematics poor foundations in number sense become glaring problems in algebra.

Schwartz and Kenney lay out numerous examples of how to build on elementary understandings to help make better sense of higher math. They focus on helping students recognize patterns and apply those concepts generally. They suggest working on these strategies as part of a deliberate agenda as a transition from elementary school to high school. They focus on using strategies that allow students to model and formulate, transform and manipulate, infer and draw conclusions, and communicate this effectively in a meaningful way. Schwartz and Kenney designed many activities that utilize the strategies and ideas behind number talks (Schwartz and Kenney, 2012).

In the book, *Extending Children's Mathematics* (Empson & Levi, 2011), the authors provide a very compelling case to include discussion and reasoning in mathematics instruction. As the authors say, "Focusing on children's thinking as we have described… can help you teach so that children understand mathematics in a lasting, deep, and interconnected way" (p. 229). Number talks, done correctly and well, do exactly that. It enables the teacher to see students' thinking in real time and foster a discussion around reasoning.

Moving from arithmetic to algebra is a struggle for many students. Understanding how to apply basic arithmetic ideas in elementary school math to generalized ideas of mathematics in high school algebra is often difficult for students. Carpenter, Franke, and Levi write about how to make that transition easier in their book *Thinking Mathematically* (2003). Carpenter et al. lay out a program to help students move from an arithmetic outlook on mathematics and use the students' understanding and reasoning to grow that knowledge into a more nuanced understanding of mathematics. They also lay out some guidelines to help develop mathematical thinking: "engage students in discussion about the appropriate use of the equals sign, encourage students to use relational thinking, foster students' reliance on fundamental mathematical properties when learning number facts, place value, and other basic arithmetic concepts, and help students generate conjectures" (p. 134). All of these guidelines can easily be integrated into number talks. Even though the students involved in this intervention have already moved into (and in some cases through) high school, many of them still lack in basic understandings. Tailoring number talks to touch on these issues can make for greater understanding of and easier fluency with mathematical concepts.

As stated by Humphreys and Parker (2015) and Parrish (2010,2011, and 2016), number talks are a way to get students involved in the thinking behind what they are doing. By following their processes you can open up the concepts behind and the processes of mathematics. Harris' research (2011) also supports the idea of open discussion of common topics in mathematics. Harris supports the strategy of having students do work and then discuss their methods in order to help build a better understanding of number sense. Schwartz and Kenney point out that recognizing patterns is a key to successfully moving from elementary math to high school math, more specifically algebra. Number talks are a good way to get students thinking about patterns and how the strategies they find while doing these talks can translate more broadly to greater concepts. Empson and Levi (2011) say focusing on children's thinking can help them develop a more deep and lasting understanding of mathematics. Lastly, Carpenter et al, also state that discussion can help students move beyond procedural understanding. The next subsection deals with how number talks can positively affect students in a classroom setting.

Math Talks and Achievement. Do math talks have an impact on achievement early on? In Susperreguy's dissertation (2013), the effects of math talks on early learners were studied. The question was to show if such talks had any impact on later achievement. Through studying 40 different families where varying levels of math talk occurred on a regular basis, Susperreguy was able to judge if a correlation existed. The results showed that quality "math talk" at home could improve a child's later abilities significantly, enough that further study was recommended. Just like reading skills, the math skills that are developed early on can give kids a leg up later on in their academic studies. Although there were limitations, Susperreguy's results showed that math talk could begin early and have significant impact on academic proficiency (Susperreguy, 2013).

As this capstone focuses on students who have already established mathematical ideas it is worthwhile to examine school-age students. With that in mind, here is an examination of the impact of a six-week math talk intervention with sixth graders in an urban school district. Since the population in this capstone is from one of the largest school districts in Minnesota, which pulls students from both urban and rural areas, as well as everything in between, it seems cogent to include a discussion of the effect of number talks in an urban setting. The authors' research approach is similar to the one that will be outlined in chapter three, which also shows the success of the premise (Okamoto, 2015).

Okamoto sought to see if an intervention could produce a change in student progress and ability to perform in their classes. Along with a long rationale for why number sense is important, Okamoto also shows the results from the intervention. A key finding from Okamoto's research is to use these talks for fluency and not worry so much about a specific purpose. That aside, Okamoto showed a strong correlation between the talks and student improvement and fluency with math (Okamoto, 2015).

Washington (2015) did a similar study but made the study population larger and tracked results at a large district in Texas across grades three through eight. Washington picked a district where number talks were already part of the curriculum and used a mixed methods approach to see if the strategy had an impact on student scores on the STAAR test in Texas. There was also some qualitative analysis of teachers and administrators to see if the effect could be felt outside of the classroom and math class specifically. In general, Washington saw positive outcomes from the qualitative analysis and slightly positive results from the quantitative analysis. Washington felt that inconsistent application of the strategy and the sample size had a significant impact on the qualitative aspect of the research (Washington, 2015).

Implementation was an issue for both Washington and Okamoto. Understanding your environment and having control over how the research is conducted and implemented is a major concern to making sure the implementation of a number talk strategy succeeds. All of the studies had problems with inconsistent implementation; however, all of the studies stated that improvement was noticeable and in general there was improvement in student achievement. The next section contains a discussion on how fractions, decimals and percentages are taught and the strategies used for moving students from additive reasoning to proportional and multiplicative thinking.

Teaching and Learning Fractions, Decimals, and Percentages

To inform how number talks for this intervention will assist student understanding, it is important to discuss the literature about how fractions, decimals, and percentages are taught in earlier grades and how those strategies can inform on what the structure of these talks will take. Susan Lamon in *Teaching Fractions and Ratios for Understanding* (2008) explains strategies for moving students from an additive understanding of these relationships to a more complete, and conceptual, understanding of these concepts in proportional thinking.

Lamon almost takes a number talk approach with the reader in this book. By engaging the reader to not only read how the students are thinking about the problem, but also try to come up with a strategy on their own. This encourages the readers to examine their own thinking and reasoning and compare it to what the students say. This creates a dialog between the reader and the author to understand the thinking behind the solution. It is this thinking that is the key to understanding what Lamon is trying to do with this guide (Lamon, 2008).

Lamon attempts to redefine how fractions are taught to students. Using the thinking behind rational number relationships and pushing a view that encourages students to experiment and formulate methods on their own gives them ownership and a method to understand the concept on their terms and in their own way. Lamon has five “nodes” about which rational number education should revolve: part-whole comparisons with unitizing, quotients, operators, measures, and ratios. By encouraging growth in all five of these concepts students are encouraged to understand the hows of fractions as well as the whys (Lamon, 2008).

In essence, if a teacher wants to increase proficiency in fractions, that teacher should use these strategies in planning lessons to help develop these abilities. Lamon (2008) does not state it explicitly, but there is an implication that these problems can and should involve discussion about strategies and methods. This model enables the student to create and use their own strategies to solve problems involving rational numbers (Lamon, 2008).

Empson and Levi also inform on how to instruct rational numbers in their book, *Extending Children’s Mathematics* (2011). They recommend starting with problems that involve equal sharing to get kids to understand the basics of fractions and rational numbers. After that, they encourage the instructor to focus on the progression of meaning, which is to say the instructor should focus on what a fraction is: “a number whose value is determined by the *multiplicative relationship* between the numerator and the denominator” (p. XXII). Next, use fractions to develop algebraic thinking and lastly show how they relate to decimals. Empson and Levi provide numerous examples of how to bring this strategy into the classroom through the book.

Educators ignore what students bring into the classroom at their own peril, according to Smith (2002). Students have numerous experiences with fractions and rational numbers long before they are "formally" introduced to them in the classroom. Educators should be using the knowledge to help foster understanding with their students in the classroom. Enabling them to bring their own experiences with this topic can help foster a move to relational and proportional thinking, allowing them to make connections and inferences which connect what they learn to what they already know (Smith, 2002).

When it comes to teaching fractions, decimals, and percentages Lamon (2008) states a clear path to helping students understand the concept and attain mastery. This process involves moving students from a procedural knowledge to a more powerful and complete understanding of the concepts illustrated in proportional thinking about these concepts. Allowing the students to "discover" the relationships between fractions, decimals, and percentages with rational numbers is the key to helping a student attain that knowledge. Empson and Levi (2011) focus on the multiplicative thinking to bring about a definition of fractions which helps students move to a more nuanced understanding of the relationship between numerator and denominator. Lastly, Smith (2002), focused on using students' own understanding of these concepts, as they see them before a mathematical definition, to help them come to a greater understanding of the concept.

Alternative Schools and Pedagogy

Alternative schools[2] are everywhere these days. There are many teachers doing wonderful and impactful things in these schools. According to Edwards (2013), there are

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[2] An alternative school is an elementary or secondary school with a nontraditional curriculum (Merriam-Webster https://www.merriam-webster.com/dictionary/alternative%20school accessed on 7-12-17)

four things needed when designing an alternative school's program: 1) to assess student needs, 2) the overall design of the facility, 3) the implementation, and 4) a drive to continuously improve the student experience. Although Edwards' dissertation focuses mostly on the design aspect of an alternative school, the insight on focusing on student needs is an important consideration.

What is the purpose of an alternative school if not to help students succeed? Foremost in that discussion should be a strong idea on what those potential students need and where in their learning trajectory the students are. This was the reasoning behind the recent split of alternative high schools in the district in which I teach. There were two populations with different needs. The more "traditional" high school-aged students needed a place where they could get the attention they needed to finish their high school requirements on time. The other population had students who were past their graduation year and really need to focus on finishing their diploma and making a plan for what came afterward. With that in mind, the district decided to "split" the services to this broader population into two distinct schools with a common purpose; helping students succeed while focusing on the specific needs of these two different, yet similar, populations. This is a strategy that follows what Edwards (2013) had in mind when designing an alternative site, a focus on what the students need and a willingness to do what works best for those students.

Faessler (2014) makes a strong case for the necessity of alternative schools. Although regular high schools work well for many, there is a small minority that has problems being successful in that setting. As education is a public good, it is imperative that an alternative exists for those students who do not fit the mold of the more mainstream setting. Faessler also states that these alternative schools implement new instruction in a more differentiated setting that allows student success in areas where that student may not have been successful before. The need is also growing: in 1987 roughly 1% of students in the US attended an alternative school; in 2010 that number had grown to 2.25%. These schools are meeting a need that is unmet in the more traditional school and in this ever-evolving society. The need for a high school diploma, coupled with some form of post-secondary education, is more important than ever.

Farrelly (2014) in some ways condemns, and in others applauds, alternative schools for the work that they do. Farrelly always makes a case for their necessity. Interviews with students who attended these schools show a powerful need for alternative pathways to a diploma for many students. However, Farrelly does make a compelling case for more accountability and higher standards when working with all students, even those labeled “at-risk”. Even though these schools serve traditionally underserved populations, maintaining high standards and teaching to prepare these students for success is vital (Farrelly, 2014).

Engaging lessons are only half the battle. Farrelly, rightly, says that alternative schools do a great job connecting with students who are typically ignored in a more traditional setting. However, these schools need to be about more than connecting with these students; bottom line, school is for education and a student with a diploma from one of these schools should have the same preparedness that a student from any other school should have. Accordingly, these schools should be both a refuge for at-risk youth and a place where they can learn and be prepared for what comes next (Farrelly, 2014).

Students leave the traditional schools for a variety of reasons, but Means (2015) came up with five main reasons for students to drop out: boredom, disconnection, lack of challenge, need for employment, and family issues. Extra social services allow for the support the students need when going through difficult times. Additionally, alternative schools have the flexibility to allow students to attend while still providing a means to deal with whatever issues brought them there in the first place (Means, 2015).

A strong case for alternative schools can also be made from a social justice standpoint. These schools historically serve underserved populations and allow for an alternative pathway to a diploma outside of the GED (General Education Degree) route. Helping these students succeed gives them a taste for success and sets them on a positive path. Creating this community as a partnership between the students and the staff can be transformative (Greene, 2015).

Creation and implementation of a new curriculum is difficult in the best of circumstances, but in the case of alternative school students it is vitally important to base the curriculum off of what is needed for the students. What do these students need, not only to graduate, but also to be successful post-high school? This was the problem examined by Ross' dissertation (2014). In this case, they implemented a curriculum that was based on career paths at an alternative school, the idea being that the students would graduate with the skills and knowledge needed to be successful after high school. The alternative setting allowed for the flexibility of implementing such a system and was met with success (Ross, 2014).

Being well prepared for the workplace is meaningless if a student does not actually graduate; whether the students follow a more traditional curriculum or a more career-based curriculum it is important that the students graduate. So what are the graduation rates at alternative schools in comparison to traditional high schools? Felix (2012) did a comparison of graduation and dropout rates between alternative high school students and traditional high school students and found relatively little differences between the two in Florida. This is an important finding based on alternative school populations. This shows that students who were not finding success in a more traditional setting are finding success in an alternative setting (Felix, 2012).

The ability to show that the students are learning is also an important part of the success of these schools. Under No Child Left Behind (NCLB, 2001), many schools were required to report Annual Yearly Progress (AYP) towards improving student outcomes on standardized tests. In one case it was shown that alternative schools had marginal success increasing AYP in some cases and no success in others. When considering the populations in these schools, the results should not be surprising. The fact that they were able to show some progress on AYP in some of these schools shows the value of alternative programs (Dickens, 2011).

Improving mathematics instruction in alternative schools is very important. It is easy when dealing with this population to lower standards in order to help the students succeed. But that approach cheapens their diploma and leaves them unprepared for future education. Many teachers worry about whether their students will leave with the knowledge needed to succeed in the future. In many cases, these students have always struggled in math and it is tempting to make it easier in order to allow them a taste of success. This can have a detrimental effect on learning and as a result the teachers must make sure their standards are high enough to merit the credit students earn. After a study with pre-service teachers in an alternative setting, it was concluded that high standards and a willingness to collaborate allowed the teachers to maintain high standards and ensure the success of their students (Dunn, 2004).

Do alternative schools have a positive impact on their students? Edgar-Smith and Palmer (2015) contemplated this question in their study on alternative schools. They surveyed students at three specific times at an alternative school. They first surveyed them at entry, then at four months, and again at the end of eight months in the program. They found that students who typically had negative attitudes towards school developed positive attitudes about their alternative program. Specifically in the arena of teacher support, the students reported a significant positive perception in the school (Edgar-Smith, 2015). The smaller class sizes and increased support network had a positive effect on the students who attended these programs.

This was also supported by an analytical study done by Wilkerson, Afacan, Yan, Justin, and Datar (2016) on how alternative schools impacted attendance, referrals, suspensions, and credits earned. Wilkerson et al. found that while attendance did not significantly improve, referrals and suspensions decreased and credits earned increased. They concluded that while the school did not improve attendance, it *did* succeed in students earning credit, which is the objective of going to school. The alternative pathway to a diploma was successful in helping these students succeed and earn their diploma (Wilkerson, Afacan, Yan, Justin, and Datar, 2016).

Among the competing philosophies and strategies for reaching students it is important to remember that alternative schools serve a vital function in educating the students alienated by a more traditional approach to learning. These schools capture and educate those who drop out for a variety of reasons and serve as a lab to test new curriculum and strategies.

According to Edwards (2013), alternative schools can be a lot of things; they are created with student needs in mind. In designing the school, the administration or district can decide what is important and focus the design on that. Faessler (2014) also makes a strong case for alternative schools as a place where those who are unsuccessful in the traditional setting can find success. Faessler's statistics show a strong and growing need for alternatives for some students to earn their diploma. Farrelly (2014) states a need for these schools although also condemns them in some cases for lowering standards in order to help students succeed. High standards should be maintained to keep the diploma from being cheapened in the name of helping these students succeed. Means (2015) applauds the flexibility of alternative schools in allowing students to achieve their diploma in a supportive environment. Greene (2015) makes the case for alternative schools from a social justice standpoint as these schools serve populations that typically are underserved in American society. Felix (2012) shows that alternative schools are a good place for many students because of their ability to retain students already at risk of dropout from school by giving them a place to be successful. Dickens (2011), Dunn (2004), Edgar-Smith and Palmer (2015), and Wilkerson et al, all found positive performance and changes in attitudes of students who attended alternative schools. The last section details student engagement and its effects on student achievement.

Student Engagement and Discussion

Student engagement and discussion is important in any subject but the need for it in mathematics has often been overlooked. Mathematics is often considered a performance subject, in that students are expected to learn a task and then to perform it in order to prove they understand it. When students are asked what their role is in a mathematics classroom, they answer, “To get questions right,” which is a very different response from those who are experts in the field. Students say it is about calculations, procedures and rules; whereas experts say it is a study of patterns. Students see drudgery; experts see beauty. This dichotomy rarely exists in other disciplines and that presents a real problem with getting students on board with math (Boaler and Dweck, 2016). To paraphrase Samuil Shchatunowski, a Ukrainian mathematician, it is not the job of mathematicians to do correct arithmetic; that is the job of accountants[3]. Unfortunately for many students, correct arithmetic is the only thing they think mathematicians do.

So how does one change the common narrative of what math is to students? Boaler and Dweck first suggest getting students to change from a fixed mindset to a flexible mindset. Mindset makes a big difference in math. According to Boaler and Dweck (2016), changing a student’s mindset in math can overcome the perennial achievement decline when students reach middle school. One of the ways a teacher can help change the mindset of students is through discussions that have no connection to procedural operations. Allowing students to discuss ideas and strategies allows them to innovate and find answers on their own. Additionally, teachers can have students explain strategies among themselves. These authors believe in the five C’s of math education: curiosity, connection-making, challenge, creativity, and collaboration. These

[3] Actual quote is, “It is not the job of mathematicians… to do correct arithmetical operations. It is the job of bank accountants.” (http://www.quotegarden.com/math.html accessed on 7-12-17)

collaborative lessons help build student skills and encourage students to be creative and innovative in how they approach and solve these tasks (Boaler and Dweck, 2016).

Discussion is a key piece to creating rich mathematical tasks; give the students a question and then encourage discussion and see where that progress leads. When students are given a problem to solve *before* discussing the subject it allows the students to fully engage their own ideas and thoughts on how to approach this concept. This engages students' natural curiosity, thus making them more open to new learning and concepts (Boaler and Dweck, 2016). In general, Boaler and Dweck advise that these tasks follow this progression: open up the task so that there are multiple methods, include inquiry opportunities, ask the problem *before* teaching the method, add a visual component and ask how they see the mathematics, extend the task to make it lower floor and higher ceiling, and, ask students to convince and reason.

The Committee on How People Learn (CHPL) in the book *How Students Learn* (2005) discusses several ways to approach teaching and learning. The one that most interests this section revolves around community-centered classrooms. The idea is that the community, in this case the classroom, develops norms that are followed much like those in a society. In a classroom that only celebrates "correct" answers, students who are unsure are hesitant to participate in any classroom activity where they might be "wrong." So the idea here is to create a classroom norm that encourages and supports students to be open about their answers, even if those answers are incorrect. This relates to the previous book where mistakes are used as opportunities to learn deeper and better (CHPL, 2005).

The mantra from math teachers often is, "Show your work," to which students readily reply, "Why, so long as I get the correct answer?" The problem that resonates here is that math teachers need to understand student thinking to make sure they truly understand the material. One of the ways in which teachers can encourage and see student thinking is through math talk. This is where a teacher has students solve a problem and then explain how and why they got their answer. This is a powerful method to model student thinking and it encourages students to actually understand why their answer is correct. This is a concept that then allows the teacher to expand upon these ideas and move into new understandings (CHPL, 2005).

When developing these lessons, it is important to build upon the strengths and knowledge of the students. According to CHPL, building on children's current understandings is a key to creating successful students and mathematicians. When engaging students in discussion it is important to start with something they already know and then lead them to the new material. You can do this using student discussion. This engages the philosophy of Vygotsky in allowing the students to act as each other's bridge to new knowledge (CHPL, 2005). The next book discusses, in detail, the effect of student engagement and discussion on student learning.

John Hattie (2012) is an educator who has spent years measuring the effect of teacher strategies on the ability of students to retain and learn. Teachers have a main impact on student learning, but what strategies work the best for all students? Hattie collected data on numerous aspects of teaching and worked out what strategies have the largest impact on the students. Hattie's information comes from a meta-analysis of 800 research topics, which, in turn, analyzed over 50,000 studies. What Hattie was concerned with was identifying what really impacts student learning and what teachers can do to improve their teaching (Hattie, 2012).

What Hattie was trying to do was tell the teachers, who often have many methods for increasing student achievement, what has the greatest impact on student learning. Hattie lays out that teachers are the *most* powerful influence on learning but only if they are caring, can differentiate the material, are clear in their expectations about what the students are learning, can relate the material to multiple ideas, and, lastly, have the support of administrators who support the teachers in their efforts to innovate and try new ideas in a safe, supportive environment. Experienced teachers focus on student engagement with the content, emphasize problem solving and reasoning, impart new knowledge and monitor that progress, and provide appropriate and *timely* feedback (Hattie, 2012). The best part of discussion is that it feeds into all of these high value strategies to connect with and teach students. In the case of classroom discussion, the power effect is d=0.82, which means that classroom discussion has a large effect on student achievement.

To initiate these discussions it is important to design tasks that create discussion on difficult ideas that push the students to new knowledge. While struggling with these problems students can discuss strategies amongst themselves that aid in collaboration and encourage groups to help each other out (Okamoto, 2015). In reading the outcomes of Okamoto's research it should also be noted that using these talks as formative assessments is a great idea to ensure that teachers keep track of learning.

Hand in hand with Okamoto's research, it is important to look to other methods for engagement. In Koelner-Clark and Newton's (2003) research they found that using rich modeling activities has a significant impact on student engagement and discussion (Koellner-Clark and Newton, 2003). So, it should go unsaid that the questions used should be ones that students find engaging and interesting.

Lastly, according to Schussler (2009), these discussions need to follow a few specific paths. Discussions should have opportunities for success, be flexible, and respect student experiences. The discussions should be difficult, but not impossible, concepts. This will allow students who easily pick up material to challenge their intellect, yet also give students who need more help a chance to succeed. This is a tough line to balance but it is vitally important when it comes to creating engaging discussions.

Boaler and Dweck (2015), state that discussions with students in math classes can help move students from a fixed mindset in regards to math. Allowing for and embracing mistakes helps students overcome their anxiety and fear of math and discussion is a good strategy to foster that strategy. The CHPL (2005) also support a “community-centered” classroom to help students feel comfortable and willing to contribute to discussion; allowing students to make mistakes in a consequence-free environment promotes understanding and individual achievement. Hattie’s (2012) work also strongly supports discussion in the classroom to help students foster a greater understanding of the material. Okamoto (2015), Koelner-Clark and Newton (2003), and Schussler (2009) all discuss how discussion can and should be used to help foster engagement and understanding in the classroom.

Conclusion

In reviewing the literature, we can see that number talks are still a relatively new phenomenon and that they are worth studying. Number talks are a great way to engage students and get them talking about math. Allowing the students to experiment and innovate in a safe, open environment allows the students to start seeing math as a search for patterns and less as a task to be completed (Humphreys & Parker, 2015;Parrish, 2010 and 2016). Alternative schools provide an excellent setting to test the effectiveness of number talks for a variety of reasons. These schools deal with a population that struggles with a variety of issues and alternative schools are a great path for these students to finish high school requirements (Means, 2015). The school setting's flexibility allows the adaptation of new curriculum with its focus on giving the students what they need (Edwards, 2013). Also, Wilkerson et al. showed that these schools help a population that, even after still struggling with attendance, can still earn credit and their diplomas (Wikerson et al, 2016). After reviewing both what Boaler and Dweck state and the research that Hattie completed, we can see that engagement and discussion are powerful strategies for success, particularly in math classes (Boaler & Dweck, 2016; Hattie, 2012). The next chapter will detail the methods used and the questions asked in determining whether or not number talks are effective in helping alternative high school students succeed in math.

CHAPTER THREE

Methods

As a result of some of the dissertations I read for the literature review, my views regarding the experimental procedure for this capstone were reinforced. The question *"Can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?"* was best answered with a mixed methods approach to data gathering (Creswell, 2014). Both Okamoto (2015) and Washington (2015) used a mixed-methods approach to gathering data for similar experiments, thus influencing the design of this study. I collected both quantitative data from student pre- and post-tests and qualitative data from student self-reflection at the end of the trimester. This would give an accurate reflection of whether or not the number talks were successful with the students. See the appendix for the test and survey used in determining the efficacy of these talks.

Rationale

Okamoto (2015) and Washington (2015) both used state mandated tests to determine the efficacy of their interventions. Since this research question is much more narrowly focused, I used a pre- and post-test that I created in order to better measure the efficacy of the strategy. The items on the test were chosen as a result of experience with this population and my experience of seeing where misconceptions occur. The qualitative portion was a questionnaire designed to encourage self-reflection by the students before and after engaging in the number talk intervention; in conjunction with the pre- and post-tests this would give me a baseline to determine growth. The point here was to see if the students felt the intervention was worthwhile for them and helped them better understand these concepts.

Pursuing this intervention during the first nine weeks of the third trimester worked best for this capstone because the schools are Alternative High Schools with a highly mobile and unreliable population. These students have typically struggled with school and as a result are reluctant learners in the extreme. Through studying attendance rates at the target schools I found that these students have the best attendance rates during this period of the third trimester, which increased the probability that they would attend during the math talks. Also these students have the best attendance during the middle of the week so I tried to do a minimum of one talk during the middle days of the week.

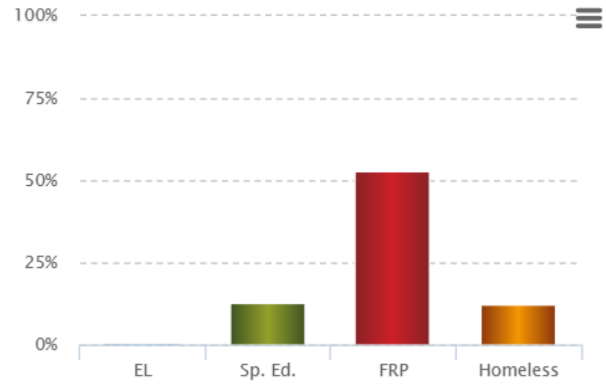
Population

The sites used for this intervention are actually two separate campuses within the same program. The first campus deals with more traditional high school-aged students. This campus has students from tenth to twelfth grades. The second campus is devoted to students who have passed their grade year; the staff refers to these students as "super seniors." This is the only site within the state that caters exclusively to this population. It is a unique opportunity for these students as the site is within a technical college, which gives many options for careers after they finish.

Two school sites were included in the study. These schools serve as the alternative high schools for a large suburban district of a large metropolitan district in the Upper-Midwest. It is the largest school district in the state and runs the gamut of urban to rural in its diversity and population. One site serves students in grades 10-12, the other, located within a technical community college, serves students ages 18-21 who have not completed high school before age 18. The population numbers across both campuses totals around 300 total students. There are roughly 30 full-time teachers on the staff, with three administrators. The students also deal with a host of mental and social issues so both campuses have support staff to help students with a variety of needs, from social workers to a therapist and a psychologist.

Figures 1-4 provide descriptive data on the student populations of each of the two sites. The data indicates that at each site 53-55% of students are on free and reduced lunch. Additionally, 12-15% of the students at both sites deal with issues of homelessness. Also, 33-45% of both sites consist of students of color. These percentages are higher than the district averages. Few students in these two programs are identified as English Language Learners. Lastly, the number of male students outnumbers that of the female students. In general, the ratio is about 65/35% in regards to male/female population.

2016 Enrollment by Special Population

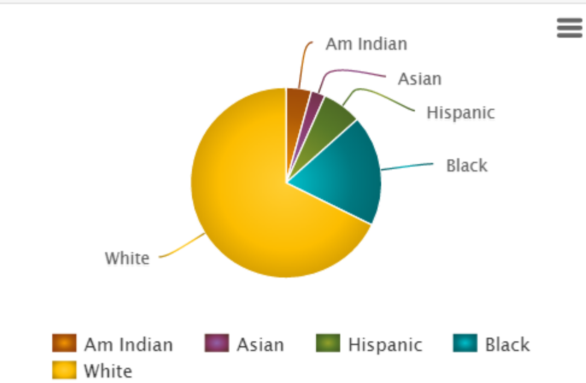


	Count	Percent
English Learner	1	0.6%
Special Education	21	12.8%
Free/Reduced Priced Lunch	87	53.0%
Homeless	20	12.2%

School Contact Information

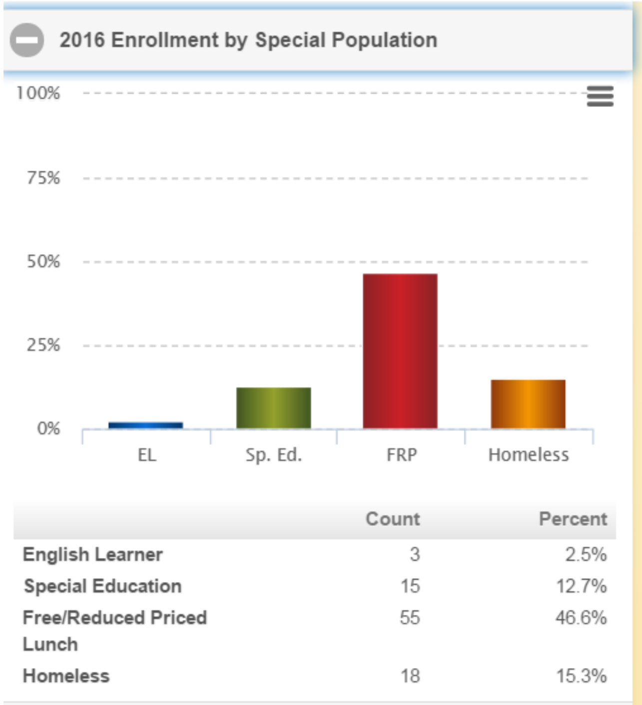
Figure 1: Breakdown of the 10-12 campus population, economic (Source, State Department of Education (SDE))

2016 Enrollment by Ethnicity



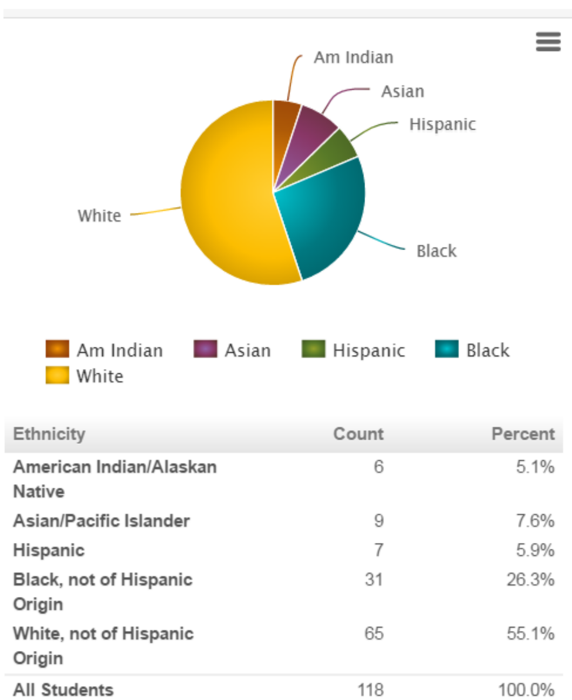
Ethnicity	Count	Percent
American Indian/Alaskan Native	7	4.3%
Asian/Pacific Islander	4	2.4%
Hispanic	11	6.7%
Black, not of Hispanic Origin	31	18.9%
White, not of Hispanic Origin	111	67.7%
All Students	164	100.0%

Figure 2: Breakdown of 10-12 campus population, ethnic (SDE)



	Count	Percent
English Learner	3	2.5%
Special Education	15	12.7%
Free/Reduced Priced Lunch	55	46.6%
Homeless	18	15.3%

Figure 3: Breakdown of Technical Campus population, economic (SDE)



Ethnicity	Count	Percent
American Indian/Alaskan Native	6	5.1%
Asian/Pacific Islander	9	7.6%
Hispanic	7	5.9%
Black, not of Hispanic Origin	31	26.3%
White, not of Hispanic Origin	65	55.1%
All Students	118	100.0%

Figure 4: Breakdown of Technical Campus population, ethnic (SDE)

The specific students who were engaged in the intervention were students who were enrolled in the Probability and Statistics course at both campuses. This class is not unique to a specific grade level; it is a required course for students to graduate from this school district. This ensured a heterogeneous mix of student ages, backgrounds, and gender. At the high school site, there had been six students enrolled in the class at the beginning of the trimester; by the end there were only four. This was in contrast to the numbers at the Technical Campus: over the course of the trimester there had been 40 students enrolled in both classes (roughly even numbers in both); by the end of the trimester those numbers had dropped to 20 total students in both classes. As students finish their credits at the super senior campus, the students were allowed to leave when they finished the credits while others were enrolled. This happened every three weeks throughout the trimester. Since this course dealt with a lot of fractions, decimals and percentage use, I determined that this class was the best place to test the efficacy of this intervention.

Process

Specifically, five types of data were collected during the intervention. The data collected was both qualitative and quantitative in nature. Pre- and post-tests were given at the beginning and the end of the intervention. There were two types of data collected that were a mix of both quantitative and qualitative data: observational data collected by a district observer, and student pre- and post-reflection on fractions, decimals and percentages. Lastly, there were two types of singly qualitative data collected: the teacher's personal journal of the observation, and the teacher- and student-generated artifacts collected throughout the intervention. (See Table 1.)

The pre- and post-tests were used specifically to measure growth of the students during the intervention. (See Appendix A for the specific test used.) After the tests were proctored the teacher engaged in the intervention. At the end of the proscribed time, the teacher retested the students on the concepts covered in the number talks and compared each student's post-test to their pre-test to measure what growth occurred.

Over the course of the intervention, the teacher was observed by the same outside observer to capture how well the students were engaged during the lesson. As number talks are a method that needs to establish norms and procedures, the teacher was observed a total of eight times by the observer to measure if students were more or less engaged as the intervention continued. The observer was given specific "look-fors" and an observation sheet to record student behavior during the lesson. (See Appendix D for the form used.) These observations occurred in the same class period at the same site to allow the observer to watch a specific set of students.

Student reflection was an important piece of this intervention as it was necessary to measure how the students felt about what they learned. A form was given to the students to measure how they felt about fractions, decimals and percentages (See Appendices B and C.) before and after the intervention to see whether the students believed the intervention was successful. At the end of the intervention the student reflections were compared to measure growth in terms of student thinking.

During the intervention the teacher also kept a journal. The purpose of this journal was to describe the successes and opportunities for better development as the number talks continued. As this is a reflection, it also gave the teacher the chance to record what he thought was significant learning and talk during the lesson.

Lastly, the teacher gathered both student and teacher artifacts generated during the intervention. The teacher used a software application called Explain Everything (2017) multiple times throughout the intervention to model student thinking to the rest of the class. These artifacts were essential to explain the reasoning behind what the students, and the teacher,

Type of Data Collection	**How the data will be used**	**When this data will be collected**	**How this data will be analyzed**
Pre- and post-Tests	To measure growth from the beginning to the end of the intervention	One test at the beginning of the intervention and one test at the end of the intervention for each student in the classes studied	A one to one comparison for each student from the beginning to the end to measure the effectiveness of the intervention as a measure of content learned, quantitative data; aggregate mean scores will also be analyzed
Observational data from a third party observer	To measure student engagement	The schedule of the schools being used fits nicely into 8 observations weekly throughout the intervention	Frequency data from observation protocol will be used to determine shifts in engagement over time; quantitative & qualitative data
Teacher reflection journal	To capture teacher notes on each number talk, reflect upon what worked and what did not. To capture teacher comments on student interactions.	After every number talk during the intervention	Analyzed for shifts in instructional decision-making based upon daily engagement with students as well as to report on significant insights during specific number talks; qualitative data
Teacher & Student Generated Artifacts	To capture in-the-moment teacher and student artifacts collected through Explain Everything	When necessary to highlight student thinking as it relates to number talk	Analyzed to show specific examples of student thinking and how it changes during the intervention; qualitative data
Student pre- and post-reflection	To measure the students' reflections on personal growth as a result of the intervention	At the beginning and the end of the intervention	Analyzed to capture students' changed perceptions as a result of the intervention; quantitative & qualitative data

Table 1: Data Collection types and explanations

were doing. As the philosophy of a number talk is to elicit student thinking, artifacts are essential in capturing this thinking. Additionally, this material was used as a supplement to the teacher's journal to better detail each lesson.

After creating and getting permissions from both the district and Hamline, the teacher distributed parental and student consent forms. (See Appendix E, depending on the student, consent was needed either from the student or a parent/guardian.) No student scores were shared with anyone other than the teacher and only the score comparisons were used for the purposes of this capstone to measure growth. The same standards were used in regards to student reflections and teacher journals. In every case, only the results were cogent to this capstone.

Conclusion

Analysis of the data collected through the above mentioned sources will be the primary focus of chapter four and answer the question, "Can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?"

CHAPTER FOUR

Data Analysis

The data collected to answer the question, *"Can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?"* follows in this chapter. The material is presented linearly as the trimester progressed.

The data was collected during the third trimester of the school year. As stated earlier, the classes involved with this study were two classes in the program at the technical college site and one class at the high school site. At the high school site, six students began the trimester course but by the end there were only four remaining. This was in contrast to the numbers at the Technical Campus. Over the course of the trimester there were roughly 40 students enrolled in both classes. By the end of the trimester those numbers had dropped to 20 total students between both classes. The end of the school year also created two problems, which are further discussed in the limitations section of Chapter Five. The first being that as the school year comes to a close, students at these schools typically stop coming to school, which is what happened here. Additionally, getting informed consent from students who typically have a lot of social and emotional issues means that those who stay in school until the end may not get their consent forms from their parents. Thus, at the high school site the data from only two students was used in this study. At the technical college site, only 28 students data are included in this study.

Pretest Data and Self Reflection Data

To judge whether the math talks had an affect on student performance there needed to be a baseline. Additionally, there needed to be a way for the students to check and see if they had any growth in regards to the material. A pretest was given at the outset of the trimester as well as when the students transitioned into the class mid-trimester. The students were also given an inventory to rate their feelings about fractions, decimals, percentages and the connection between the three concepts.

The pretest results were not surprising as they reinforced the knowledge that students struggled with these ideas. After the pretests were given and the scores recorded as percentages, the scores were averaged according to section. The students on average scored 19% on the fractions, 30% on decimals, and 15% on percentages for accuracy. A majority of the students also rated themselves at the basic level of understanding of these concepts and many did not understand how all three of these ideas were related. (See Appendices A and B, respectively, for the pretest and the survey.) The results of the pretests and the self-reflection portion were used to help formulate math talks that would help the students' progress on their understanding of these ideas.

Math Talks

The math talks were given over a nine-week period in the last trimester of the school year at the target schools. As a result of time constraints, there were only ten math talks over the nine-week period. The intent was to do two a week, but that was not possible given the needs of the curriculum and the class. The outside observer visited the same class period at the same school throughout the intervention; in this case it was the first hour class at the Technical College site. She was able to make eight observations and those observations will be integrated into the discussion that follows as they occurred with the math talks. The observations are included to add context and to back up the recollections of the teacher doing the intervention. The math talks were given to three separate classes, and the data below represent discussions and answers from all three classes in chronological order. Where the observation comes from the outside observer it is noted in the text. This section is broken into three subsections, the first three weeks, the middle three weeks, and the final three weeks.

The First Three Weeks. The first math talk was conducted without data from the pretests and was a question about fractions. The decision was to do a baseline discussion on elementary fraction addition. The first math talk was on adding the same fraction, specifically 6/7, 16 times and forming a quick discussion about this operation. After the discussions and reflecting on what was discussed, it seemed most of the students had forgotten the basic rules of fraction addition. (See Figure 5.) The most

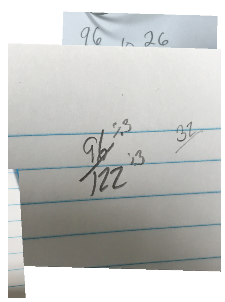


Figure 5: Student A1's work

common answer seen while circulating amongst the students was 96/122. It should be noted that the students were not allowed calculators during these talks so some of their arithmetic was off; for example, seven times sixteen is 112. This is perhaps the most common mistake seen involving fraction addition. Additionally, if you notice the picture, this student was attempting to simplify the fraction. The student guessed incorrectly that both numbers were divisible by three and was attempting to put the fraction into its simplified form. The student would have found, upon finishing his simplification that this fraction is equivalent to 6/7. (See Figure 6.) This was the natural progression reached

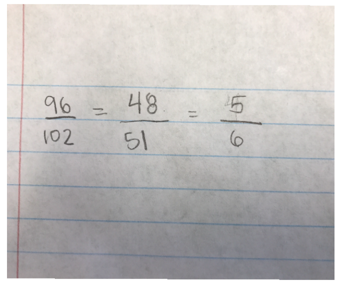


Figure 6: Student A2's work

during this discussion and it led to students wondering what went wrong. How could they add a fraction to itself 16 times and get the same answer? This misunderstanding comes from how students comprehend fractions and addition. Following the basic rules of addition, this is the answer you should get. However, this shows an inadequate understanding of a fraction as part of a whole. Upon seeing this result, the question was turned back to the students: "What is 6/7 close to?" The student noticed that it is pretty close to one, so then the answer should be close to 16. As Lamon (2008) points out, this is clear proof that the student understands additive thinking but has not yet progressed to proportional thinking.

The next student attempted to look at the problem visually and, in a smaller form. (See Figure 7.) This student was attempting to visualize how to rearrange six parts of

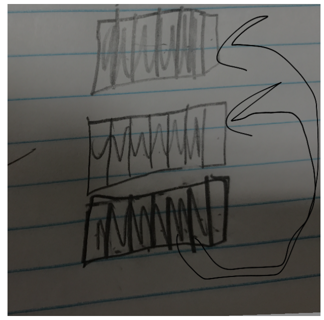


Figure 7: Student A3's work

seven into a picture and then expand that thinking to 16 of the boxes. This student understood that the fraction was a part of a whole, however, he still did not complete his calculations in the time allotted. Most likely this student would have reached the correct answer if he had more time. This is a move towards proportional thinking but this understanding is at best progressing.

Attempting to determine where the students were in regards to their knowledge and thinking of fractions, decimals, and percentages, the next math talk was a question with decimals. The question seemed easy enough: "What is the answer to .75 times 33?" This was picked because the number 33 is not a factor of four but is close to it. This would require the student to contemplate how to arrive at the answer without an easy means. Naturally, the leap to whether or not the number 33 was a factor of four would be a good indication of proportional thinking. There were a few strategies on how to figure this one out from a couple of students. First of all, some students looked at it like a monetary problem. They saw the question as, how much would they have if they had 33 groups of $0.75. The reasoning was sound and allowed the student to reach a correct answer after a lot of work. (See Figure 8.) They knew enough to know that 10 groups of $0.75 are equal to $7.50 and they knew that there would be three groups of $7.50. This

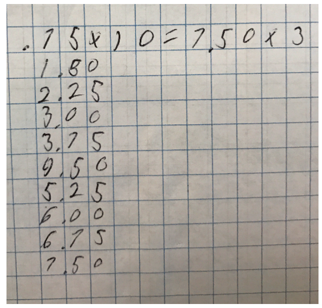


Figure 8: Student A4's work, also notice the misuse of the equals sign

led them to a starting total of $22.50 to which they only needed to add three groups of $0.75.

The next student saw the question a little differently. He converted the question to a fraction and then simplified the ensuing improper fraction into a mixed fraction for the answer. (See Figure 9.) This showed a clear understanding of the steps, and the student answered the question efficiently. What was remarkable about this is the fluidity of his

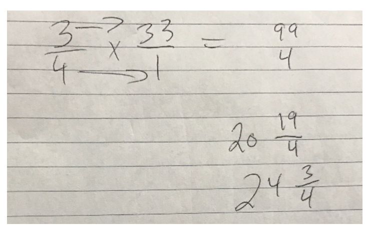


Figure 9: Student A5's work

thinking on this problem. Notice the student immediately converted .75 to ¾ and put 33 over one as they learned when first taught how to multiply fractions to whole numbers. This student then followed a linear process to get the complete answer in simplified form. As it relates to this population, I would state that this is the exception and not the norm. This particular example of student work led to a very good discussion of the fluidity between fractions, decimals and percentages. Whether or not this is an example of proportional thinking is unknown, but this student demonstrated during the ensuing discussion that he clearly understood, and could explain, his answer to this question.

This next student looked at this problem in yet a different way. He saw a math problem and applied the common algorithm to get an answer. (See Figure 10.) In this case, the student chose to use a standard multiplication algorithm to find the answer.

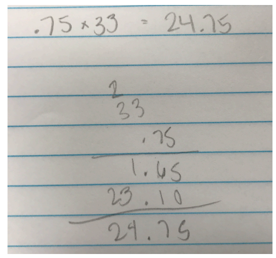


Figure 10: A2's work

This student clearly understands how to work a problem like this but I am left wondering if the student could figure this out in a different way if they did not have access to paper. Also, this method does not give any insight to how the student sees this question other than a chance to use a well-known algorithm. It works but do they understand *why*? Either way, the preceding three examples showed evidence that the students understood how to work problems such as this one. Most exceptionally, it seems putting these types of questions in terms of money was a very good way to help students visualize how this question could relate to real life situations.

The next talk used percentages. Again, this was an opportunity to see how these students viewed this concept and how well they could deal with the subject matter. This question asked what is 30% of 67.23. This was an opportunity for the students to see if they could visualize what percentages looked like and how they could use benchmark numbers and/or benchmark percentages to get an approximate, or actual answer. In this case, I had a number of students shut down. They believed that this was an impossible question if they could not use calculators. The students were asked if they could get an over/under estimate, which is asking them to find a number it had to be above and a number it had to be below. Although a discussion of benchmark numbers had come up previously, this was really the first time it was stressed as a way of getting the answer or a rough approximation of it. There was some pushback on this concept. Many of the students stated that it had to be an exact answer. This was an example of a fixed mindset in regards to what students think about math and what mathematicians know about mathematics (Boaler, 2016). Overcoming this obstacle had caused students to shut down in the past with this type of question, but once they realized that an estimate could work, they were much more inclined to try.

This was the last talk in the first three weeks and it was observed. The observer noted that a few students started using benchmarks to get ideas of where the answer might be. One student used 70 as a benchmark because it was close to the target number of 67.23 and stated that it must be below 35 since it was half of 70. (See Figure 11.)

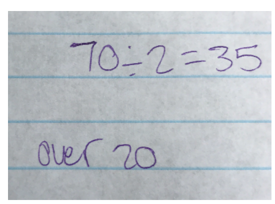


Figure 11: A6's work

As he stated, "…cause it was close to 67 and I did half cause it's close to 30% so I know it's going to be less than that." Another student (A7) recorded that 30% of 60 is 20 so it must be more than 20. Again, his arithmetic was off a little but the thinking was sound. That same student noted that 10% of 67.23 is 6.72 and then multiply that by 3 and get an answer very close to what it should be. A different student (A8) responded that thinking about the "10% thing" was a good idea. (See Figure 12.) A different student (A9) came

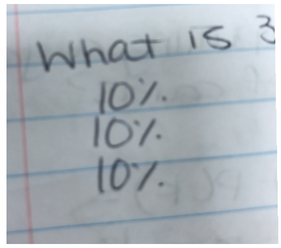


Figure 12: A8's work

up with a similar conclusion and applied it to get the answer.

The Middle Three Weeks. In the first talk for the middle three weeks the students were asked to find the product of 2/3 and 16. Again, this talk was picked to provoke the students to find a connection between the fraction and a number not divisible by the numerator. This would require the students to think about what the number means and have a strategy to find the answer while not relying on calculators. This was another opportunity for the students to use an over/under approach to find, at the very least, an estimate for the answer. Also, it was hoped that the students could start seeing fractions as an operator in addition to seeing this sort of problem as a fraction (Lamon, 2008). For example, they could see it as two times sixteen divided by three, or sixteen divided by three times two.

A common approach appeared that was reached by a few students (See Figure 13.) that worked surprisingly well for finding a good estimate of the answer. As a result

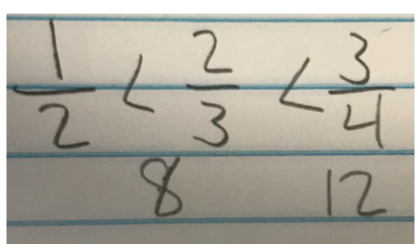


Figure 13: A3's work

of 16 *not* being divisible by 3, the students used benchmark fractions to find a range. A number of students could figure out that half of 16 is eight and that ¾ of 16 is 12, so since

2/3 is between ½ and ¾ the answer to the problem must be between eight and twelve. The outside observer noticed this discussion as well and recorded it in her notes. This led to a discussion of whether or not the answer should be closer to ½ or ¾. It is good that they were able to find an estimate, but were they then able to deduce which end of their estimate would be closer and it also gets to the heart of number order? Which fraction is 2/3 closer to, ½ or ¾? Eventually, there was a student who said that ¾ is 75% and ½ is 50% and 2/3 is close to 67%, thus they were able to deduce the answer must be closer to twelve than it is to eight. This may have been a result of growing understanding of how fractions and percents are related.

There were also students who immediately applied their knowledge of fractions to this problem and got an answer of 32/3. (See Figure 14.) A student who did this wanted

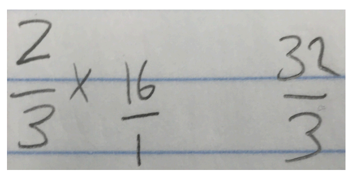


Figure 14: A1's work

to know whether or not the fraction needed to be simplified; this was turned back to the students and elicited an excellent discussion on whether or not an "improper" fraction is indeed improper. The course of the discussion played out and the answer was if you want to, simplify; if you do not want to, then follow your discretion. This is also something that is context-dependant; given the context, some representations of the answer could be better than others.

As there were successes, there were also some answers that revealed a fundamental misunderstanding of how you multiply fractions. Another student (A9) epitomized a common misunderstanding of when and how to apply cross multiplication.

In this case, the student cross-multiplied 2/3 and 16. This student did place the 16 over one but then flipped the 2/3 and after multiplying ended up with 48/2; this gave him a final answer of 24. This provoked a good discussion on whether or not this was an appropriate answer given that we were multiplying 16 by a number that was less than one. After some quick discussion, it became clear that this answer did not make sense. In a different class, another student did the same thing but this time flipped the 16 and one and arrived at the answer of 2/48. (See Figure 15.) This brought about a different

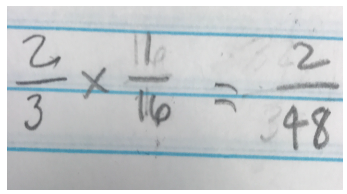


Figure 15: A10's work

discussion about whether or not this was an appropriate answer as well.

Continuing with our excellent discussions about multiplying fractions, the next talk asked the students to multiply 2¾ with 20. It was thought that the students could build upon what they did the previous week and make some new connections. This question did cause some consternation even though the previous question was similar to this one, and the intent of the question was questioned. Did it mean that we were multiplying 2 times ¾ times 20 or were we multiplying 20 times 2 and ¾? (See Figure 16.) This was a valid question and

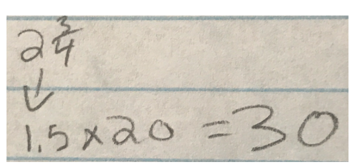


Figure 16: A5's work

it gets to the heart of the confusion many of these students suffer upon viewing this type of question. (See Figure 17.) This student had trouble understanding the intent of the

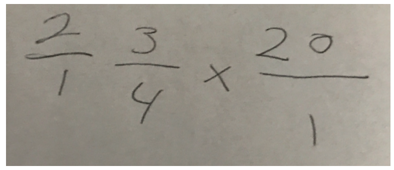


Figure 17: A9's work

question. As evidenced by the picture, the student was trying to implement a strategy, possibly one seen in the previous week. By placing the two over one along with the 20 over one he tried to put the question in a way that makes sense. This student was setting up the question in a way that would have garnered an answer of 120 over four, or 30. This is evidence of an attempt to apply a method without really understanding why that method works. This also shows a lack of understanding of the distributive property of multiplication of addition to a mixed number expression[4].

A few students also attempted to apply an over/under estimation approach to get an idea of where to start. Most students knew that the final answer would be higher than 20 because 20 was being multiplied by something larger than 1. Pushing for more information on this, most students were able to see that the answer must be greater than 40, but less than 60, based on the fact that 2 and ¾ is greater than two and less than three. Through questioning this thinking, the students were able to determine that the answer would be closer to 60 than 40 because 2 and ¾ is closer to three than it is to two.

Yet another student arrived at an answer of 40.75 because, in essence, they noticed that ¾ is added to the two so he multiplied two and 20, which is 40, and then added .75 to the 40 for his final answer. (See Figure 18.) This answer was both

[4] For example, this could be seen as 2 ¾ x 20 = (2 + ¾) x 20 = 2 x 20 + ¾ x 20

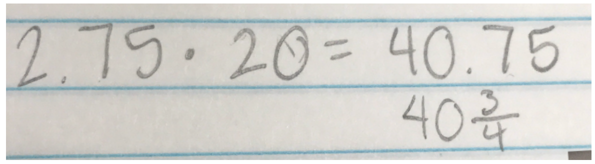


Figure 18: A2's work

remarkable in its sophistication and misunderstanding. This student, getting that 2 and ¾ is equivalent to 2.75 tried to apply this knowledge to get an answer but fails to get that he needs to multiply the .75 and 20 as well as the two. He succeeds at understanding the additive property in regards to 2 and ¾ but fails to apply this property to distribute the multiplication over the addition. Intuitively this answer makes some sense, you have an answer that is more than two times 20 but it fails to understand that the answer should be closer to three times 20. When presented to the class this answer fostered an excellent discussion on the distributive property of multiplication over addition.

There were also a number of students who understood this and managed to get the answer in a few different ways. Using the idea of the previous student, this student (A5) properly applied how to break a complex question into something easier and then brought the smaller problems back to find the correct answer. (See Figure 19.) As evidenced by

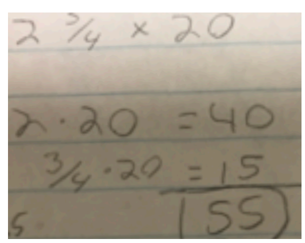


Figure 19: A5's work

this solution, the student had broken the problem into two separate (and easier) problems and then added those results to find the correct answer. This showed that the student clearly understood the distributive property of multiplication over addition.

Understanding that 2¾ is similar to adding those two together they were able to multiply both by 20 and then add the results after. This is easier for some than converting the mixed fraction to an improper fraction and then multiplying. (See Figure 20.)

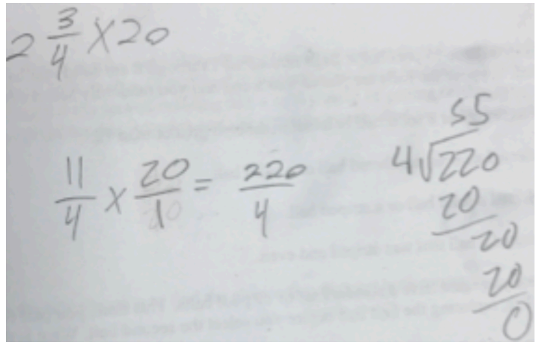


Figure 20: A10's work

As evidenced by this student's work there was a lot more procedure to doing it this way; however, both students arrived at the correct answer.

The direction of the next number talk task moved towards decimal multiplication. In this talk, the students discussed strategies and procedures to figure out the product of .47 and 720. This is another situation where a lot of students had trouble figuring out where to start without a calculator. This provided an additional opportunity where an over/under estimation strategy was discussed and encouraged. Here is an example of what one student's estimation process was. (See Figure 20.) As you can see, the student

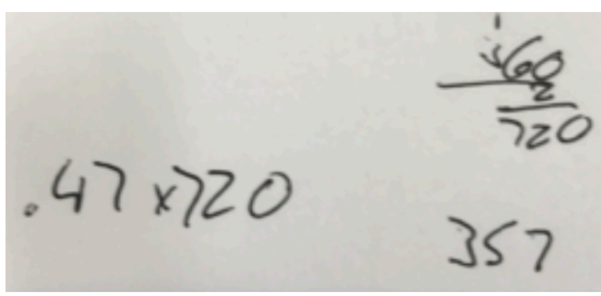


Figure 21: A11's work

rounded .47 to .5. In this case, the student decided to make the problem easier by figuring out what half of 720 is rather than following the algorithm to an answer. Since this student saw that .47 was a little less than half, the answer must be a little less than half of 720. The student's answer was a little off (by about 20), but in light of the complexity of the decimal, this was a decent estimate. There was also another student

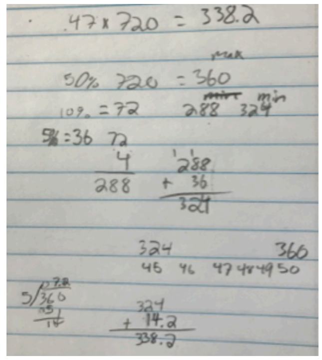


Figure 22: A5's work

(A5) who used a similar strategy to find the answer. (See Figure 22.) He not only saw that 360 was half of 720 but also decided to try to find a lower bound. Previous talks discussed finding 10% of any number and this student used that strategy to find 40% of 720. This enabled them to have both an upper and a lower bound. Looking carefully you can also see that this student then determined 1% of 720, then adding that number seven times to 288 allowed the student to, eventually, find the correct answer.

Two days later there was another talk, this time observed by the outside observer. This talk was another mixed fraction multiplication, 3 x 2 7/8. A student (A12), who had his first number talk on this day as a result of a recent schedule change, looked at the problem from a purely real world standpoint. This student was someone who enjoyed working with his hands and working on cars. The student saw the question as how much wood would be needed if it were cut into three equal parts of two and seven eighths. Recognizing that it was three times two as well as three times 7/8 this student used the distributive property to first multiply three and two and then multiply three and 7/8, i.e.

3(2+7/8)= (3×2)+(3×7/8). The student then added those two outcomes together to get the answer. (See Figure 23.)

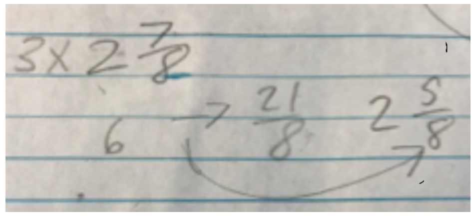


Figure 23: A12's work

The language used was very precise by this student. Once this student obtained the two answers from the multiplication, six and 21/8, he then needed to work this into an easy answer. Looking at the fraction he described needing to find "as many whole numbers as I could" thus allowing him to simplify the fraction to two and 5/8. Adding that total to six gave the student the answer. (See Figure 24.) This was a very good way to approach

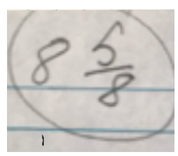


Figure 24: A12's answer

the problem and the language used by the student put the question into a scenario that allowed everyone to see the worth of this knowledge.

In the same class, another student (A5) used the over/under estimation approach on the problem. In this case, the student determined that the maximum the answer could be would be three times three since three was slightly larger than 2 7/8. Likewise, he also determined the lower range at whatever 3 x 2 ¾ would be. The rationale, put forth by this student, was that ¾ was much easier to work with than 7/8 particularly if you look at it from a monetary perspective. (See Figure 25.) As is seen in the figure, this student

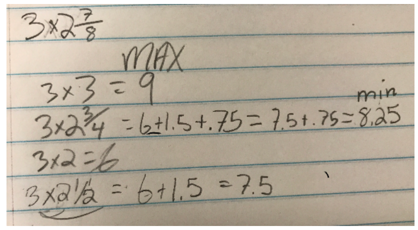


Figure 25: A5's work

figured out what half of three was and added it to six and then figured out what a quarter of three was and added it to the previous to arrive at a minimum of 8.25. So, this student had narrowed his answer down to a relatively small range, from 8.25 to nine. With a little more time to figure out how eighths fit in with fourths this student may very well have found the correct answer.

The next class period had another student (A13) who looked at this problem in yet another way. This student began by converting the mixed fraction into an improper fraction. This student altered the question to three times 23/8. Previously, there was a discussion about how multiplying improper fractions can be easier than multiplying mixed fractions for some. It should be noted once again that these students were in the midst of a unit on probability and by this time were multiplying fractions regularly. After the student had the problem set up, it was just a matter of multiplying numerators and denominators. (See Figure 26.) This student chose to leave the answer in “improper”

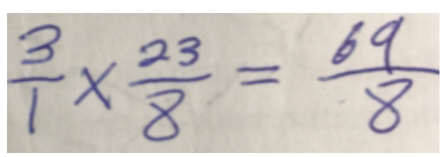


Figure 26: A13's work

form. The class had discussed in the past whether or not this was allowed and decided that yes, it is allowed.

The Final Three Weeks. The first talk in the last three weeks was a scenario about calculating tips. This was an application of the properties of percentages where the students were encouraged to find an approximate answer. The question asked the students to calculate an 18% tip for a meal costing a total of $73.42. As this was an application of something that the students knew something about, they quickly formulated a plan and managed to get a quick result. One student noted that 10% of the total was very close to $7.50; as a result they doubled $7.50 and gave the server a $15 tip. (See Figure 27.) This was a nice solution to this problem and it integrated previous

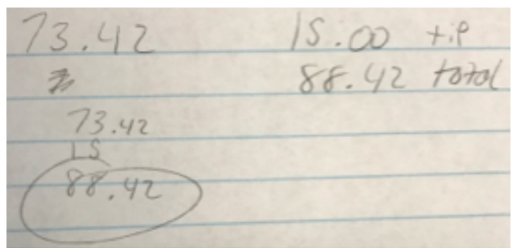


Figure 27: A12's work

concepts of estimation and benchmark numbers to find a quick and easy solution. Although it was imprecise, this gave a relatively accurate (if not generous) tip to the server and allowed the student to move on without agonizing over a total.

The next student (A8) in this class attempted greater accuracy but started to falter when trying to get to exactly 18%. (See Figure 28.) As shown here, she easily calculated

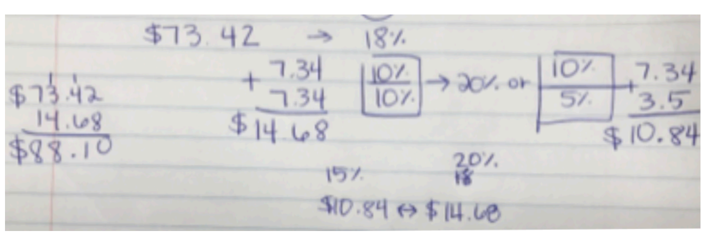


Figure 28: A8's work

what 20% would be and then did 15%. Her strategy was to eventually work her way to 18% by narrowing her answer between these two extremes. This was a sophisticated use of an over/under estimation strategy. Another student (A13), in a different class used a similar approach but subtracted 1% twice from the 20% total. (See Figure 29.) This was

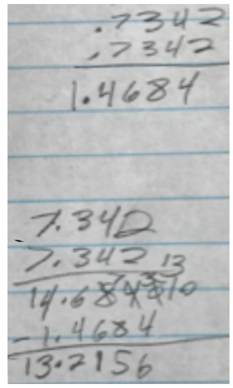


Figure 29: A13's work

a solid approach to the question and, after adding this to the bill total, would have resulted in the answer. Lastly, on this problem, there was another student who used the standard algorithm to find the tip and the total bill. (See Figure 30.) This shows that the student thoroughly understands how to use the standard algorithm for this type of calculation.

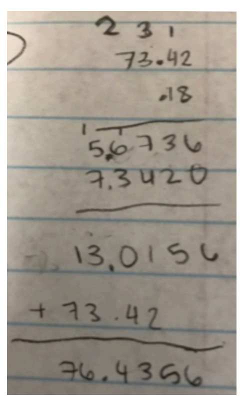


Figure 30: A14's work

The next math talk was a discussion on a consumer question. Which was a better deal, 1/3 off a particular item or buy one get the second item half off? This is a question that requires good number sense. This question really puzzled the students and forced them to think about what these numbers mean. This also spoke to the relevance of understanding fractions from a consumer's standpoint. The students' answers were not typically mathematical here, but did reflect the thought that went into their decision. The first answer showed that this student believed that the buy one get one half off was the

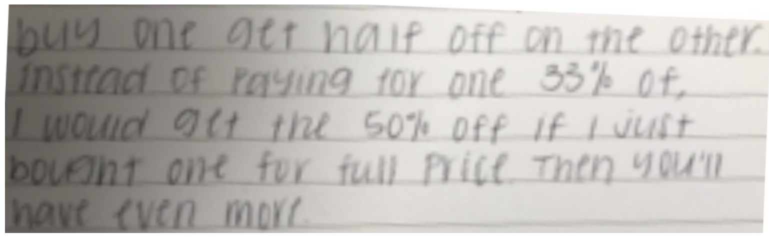


Figure 31: A8's work

better option because you get more for less. (See Figure 31.) This gets to the crux of a misconception that half off a second item is better than a third off of a single item. However, in this case the purchaser must purchase another at full cost and that is where the confusion arises. Another student (A3) attempted to answer this question visually. (See Figure 32.) After engaging in our discussion, this student stated that the buy one get one half off is in reality a 25% discount on both, which is worse than 1/3 of one item. This showed that this student was "unitizing" this question and driving at a solid

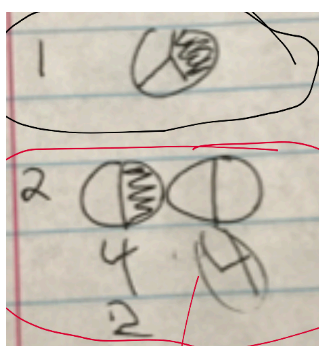


Figure 32: A3's work

understanding of this concept. The outside observer also remarked on a discussion revolving around whether or not a person needed one or two of the item in question. This spiraled into a discussion on American consumerism and what is really needed. After the resolution of this discussion, the class determined that if two were needed they would be better off buying one item, leaving, and then coming back in and purchasing another. Yet another student (A5) tried to answer this mathematically. (See Figure 33.) This student hypothesized a scenario where the item on sale cost $80. Barring minor arithmetic errors,

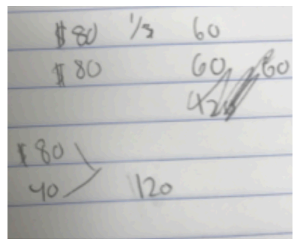


Figure 33: A5's work

this strategy was sound. Finding a number that would be divisible by both three and two would have been the best choice and this student attempted to do that. (This student should have used 90 or 60 as those would have worked better than 80.) He followed a nice procedure, which would allow him to compare the final cost of doing both activities thus giving him an answer. This was an elegant example of understanding the math behind this question and a nice way to check this sort of scenario in the real world.

The last number talk was based off a number talk in Parrish's (2016) latest book on number talks (p. 121). The idea behind this talk was to use benchmark fractions to find other percentages. The first set used ¼ to find percentage equivalents of 1/8, 5/8, and 7/8; the second set used 1/3 to find percentage equivalents of 2/3, 1/6, and 5/6. The purpose was to tie all of these concepts, fractions, decimals, and percentages and give the students a chance to use the relationships of these concepts to expand their understanding and use the strategies of the previous number talks to find more difficult percentages. The discussions during this number talk were really about how these fractions related to each other and how we can use the benchmarks to find new percentages. As the talk progressed, it was clear that the students were still struggling with the material even after a nine week number talk intervention. However, the talks had helped a few of the students and, as the discussion progressed, the students began to make connections between how the fractions related to each other and how they could use their strategies to determine the percentages based off the fractions. As with all learning techniques, the students who put the most into the talks seemed to get the most out of the talks. The last talk in the intervention tied the previous talks into a final application of the knowledge gleaned through the number talks.

In sum, the talks did a good job of engaging the students, discussions were lively, and the students seemed to see a side of mathematics they had not seen before. The outside observer also noticed a high level of engagement while the talks were in session. The students seemed willing to experiment and try new ideas in future talks to see if it could help them as they worked through the trimester. In the next section, there is a discussion on the results of the post-test and survey.

Post-test Data and Survey Results

This was the most frustrating part of the intervention. Attendance was spotty at the end of the intervention and the roster of students experienced a high level of turnover while the trimester progressed, primarily at the Technical site; the High School site had a few additions throughout the trimester but was the most stable from the beginning to the end. Of the students who took the pre-test and survey there was only one student who managed to do both. While a direct comparison, with such a low sample space, would be meaningless statistically, there will be a comparison for the one student and a comparison of general averages for before and after the intervention.

As previously stated, the average score for the students on the pre-test for fractions was 19%, decimals 30%, and percentages 15.5% in terms of accuracy. Comparing these results with the post-tests of the students who took the post-test showed some significant growth. In fractions, the students increased their scores to 47%, over double what they scored previously; decimals doubled exactly to 60%, and percentages grew the most anemically, a little less than double to 29%. The significance here is that in *all* areas there was growth at both sites as measured by this aggregate average.

Comparing the one student who took both the pre and post-tests showed significant growth as well. Initially, this student scored 50%, 60%, and 38% respectively on the pre-test on fractions, decimals, and percentages. On the post-test, this student scored 80%, 100%, and 75% respectively. Looking at this student's pre-test scores, it is easy to see that he had some ability prior to the intervention. It is possible that the intervention simply reminded him of previous knowledge he had understood and just needed some reinforcing. This is an easy conclusion to make but comparing this student's pre- and post-survey shows a student who felt he was okay with fractions but unclear on decimals, and very unreliable on percentages. At the end of the intervention, this student felt he backslid on fractions (although comparing the pre- and post-test results one could conclude this may have been a mistake), but progressed on all topics, including the connection between all of these concepts. Most importantly was the exchange on the day of the post-test. This student, whilst in the process of completing the post-test asked if he could convert the fractions to decimals and do the calculations without using fractions (see problems 5 and 6 on the pre/post-test). This was taken as elegant proof that this student had truly figured out the connections between fractions and decimals, at the very least.

The rest of the students, at the end of the intervention also showed, in general, more positive attitudes regarding these concepts. Since there was only one who did both there is no way to do a direct one-to-one comparison of the pre- and post-survey results. (See Appendices B and C, respectively.) However, the students in the post-survey showed a more positive attitude about these concepts than those who took the pre-survey. This could be another example of growth as a result of the intervention.

Conclusion

The data shown in this chapter details the pre- and post-test results as well as the results from the pre- and post-survey. The pre-tests showed that, on average, these students had trouble with all of the concepts laid out in the number talks. The data also showed how the students grew in their knowledge as a result of the number talks. The outside observer's notes detailed how the students discussed these concepts and helped explain their thinking on the material. The outside observer also noted that the engagement levels were high during the talks. Lastly, comparing aggregate data from the beginning to the end showed growth in understanding all of these topics. An examination and synthesis of the data is presented in Chapter 5.

CHAPTER FIVE

Conclusion

This study asked the question, *"Can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?"* The previous chapter detailed the results of this intervention. It also explored student work samples to show growth of understanding as the trimester progressed. The following chapter will outline the conclusions reached as a result of that data.

Limitations

In general, I believe this was a worthwhile intervention for the students involved in these classes. Any attempt to make math more relevant for our students is a worthwhile endeavor. With that said, I found the data collection portion of this capstone to be frustrating largely because of the nature and processes of alternative schools and their populations. Additionally, as the school year wound down there were more and more students who failed to show up at all for their classes. This is a common occurrence at these schools. Students understand that they will be returning in the fall and can make-up whatever they failed to finish at the end of the previous year, which was the case at both campuses.

Consistency was also a problem. These schools both add new students every three weeks and students finish and move to new classes throughout the trimester. Gathering relevant data without a serious disruption in the classroom routine with these transitions was difficult. Should the student be pulled to do their pre- or post-test and survey thus taking away class time? Or should it be done at a different time that does not disrupt their normal schedule? Doing these before or after school is also not a realistic approach as most of these students rely on transportation from the district and cannot stay after school or arrive earlier in the day.

I feel both of these concerns are apparent in the data collected. The fact that I only had one student complete both the pre- and post-test and surveys is clear that collecting quantitative data was a struggle. Also, as students were added and removed from the classes it was really difficult to proctor the pre- or the post-test so doing a direct comparison from beginning to end was almost impossible.

However, the data I did manage to collect showed a glimpse of what these talks can do if they are used regularly. In particular, I noticed a greater willingness to try on difficult problems as the trimester progressed. I also noticed that in all of the classes the students were engaged and discussing the problem at hand. This reached beyond the talks to regular classroom discussions as well. The outside observer noticed this as well as the trimester progressed. Her notes illustrated students willing to try new strategies and a desire to understand what works and what does not.

The comparison of the scores in the aggregate was also useful. Even though I was unable to do a direct one-to-one comparison of students at the beginning and at the end, I was able to compare aggregate scores from the pre-tests to the aggregate scores of the post-tests. By comparing average results, I noticed that there was significant growth between those who took the pre-test and those who took the post-test. In this case, the average score on the fraction portion of the test grew from 19% on the pre-test to 47% on the post-test. Although far from a passing grade, the students more than doubled their scores over the pre-test. I noticed similar growth in the other sections. The decimal section grew from 30% to 60% and the percentages grew from 15% to 29%. In each case, there was significant room for improvement and the students did improve.

Whether the improvement was the result of the talks or the curriculum is difficult to discern. As stated before, this was a probability and statistics class. During the first six weeks of this class there was significant focus on probabilities, which can be expressed as a fraction, decimal, or a percentage. The students were seeing these types of operations regularly. I believe, however, that these two separate things fed into each other. The students understood the material better as a result of the talks and the talks were seen as more applicable to the classroom as they used a lot of the strategies discussed during the talks as the class unfolded.

Implementation was an issue. The plan was to do two number talks a week. However, as a result of pacing and the need to cover the curriculum, it was hard to do two a week. As seen in the data in Chapter Four, there were only ten number talks in the nine-week intervention. This means that there was a little more than one a week. There would most likely have been a larger gain by the students in the pre- to post-test if there had been more talks that covered more topics. The talks barely touched on percentages and, looking at the post-test results, this showed in the aggregate data.

Scheduling was also a problem. Trying to find a day of the week that worked for both the class and the outside observer's schedule required the teacher to move some things around to make time for the talks. Although this was a moderate problem in the grand scheme of things, it still was something that got in the way of the intervention timetable. Also, the talks, which should only be a maximum of 20 minutes (Parrish 2010, 2016 and Humphreys & Ruth 2015), routinely went beyond that time. Part of this was getting the students to initially engage, and the other part was that it was tough to stop a good discussion once it got going. The discussions intrigued the students; it was really hard to change gears when what was being discussed was worthwhile to them. The problem of too many great ideas conflicting with not enough time to implement them created a shortage for the needed curriculum in this class. The statistics and probability class at this district is very expansive and getting through the class is hard enough without giving up 30-60 minutes a week to something that is tangential, yet important, to the curriculum. So, it was decided that in lieu of more frequent number talks, longer ones where the focus was on understanding and student thinking were more important.

Takeaways

So, can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students? In my opinion they do, but does the data of this intervention back that up? To a degree it does; however, it creates more questions and a need for a more comprehensive and longer study. The sample size of this study was just too small. Trying to make a conclusion for the population at large from this study would be like trying to predict a presidential election by studying one suburb of a major metropolitan area.

This study could be lengthened and increased to cover all of the students in these sites, and other alternative high schools as well, to create a larger sample size over a longer period of time to truly measure the growth as it relates to number talks (maybe over an entire school year). Anecdotally, the students had more confidence and were more willing to try new ideas after these talks than they had been in the past. The students also found math to be more interesting and applicable to their lives, or to quote a student from their post survey, “It's helped a lot in the ‘real world’ aspects of life.”

As a math teacher, these talks helped the students better understand how fractions, decimals, and percentages fit into the world of math. In my experience, many of these students have trouble with basic number theory and placement of fractions on a number line. This can be attributed to a number of things both related to and unrelated to their schooling. When given a chance to examine their understanding and an opportunity to glean why the material is important, the students strategized, discussed, and experimented with new procedures to increase their understanding of the material. Letting the students have a consequence-free environment with the support of their peers and their teacher allowed them to fearlessly make mistakes. Embracing the philosophy of Boaler (2016) allows the teacher to celebrate mistakes and allow misconceptions to be cleared as a result of routine classroom activities.

Number talks have been shown to be successful with elementary and middle school populations. The research on high school populations is almost non-existent. This may be because of how math is taught at the secondary level or just a lack of knowledge about the process. For me, learning about number talks was transformative in regards to how I approach learning in my classroom now. I rely less on direct instruction and more on student-led discovery. This has made my classroom more student-centered and more engaging. Students talk about their strategies and ideas and I have seen procedures develop which were intuitive and made sense, even if it is not the way I would have done it.

Students want to learn. They want a chance to show what they know and they want a chance to experiment with new and old knowledge to find different ways of understanding the material. Number talks, as a strategy, allow them the opportunity to flex their creative side in a subject where they normally do not. Mathematics is a beautiful discipline and we rarely allow our students to see that beauty. Frequently, all they see are procedures and rules and a quest to find the "correct" answer. Number talks pull us out of that rut; at least the talks did that for me. My classroom had stagnated and I never even saw that. I now see lessons as a chance to engage my students in thoughtful, reflective discussions about the nature of my class.

Connections to the Literature Review

I saw a number of connections to the literature review in Chapter Two with the data I collected during this intervention. Connecting what was done in these classes with the information provided by Parrish (2010, 2016) and Humphreys & Ruth (2015) was fairly easy. The observations noted by these authors were vital to the planning and implementation of this intervention. Additionally, the results echoed those found by the aforementioned authors in the course of the intervention. The discussion-centered way in which the math talks were done in the classrooms also aided the overall performance of the students in these classes, creating a dynamic and engaging way to get the students interested in mathematics.

Relevance is key to working with any student, regardless of where they attend school; the discussions were a good way to implement Boaler & Dweck's (2016) advice on overcoming fixed mindsets in students in the math classroom. Building on what Boaler & Dweck (2016) stated in their book, the environment created by the math talks allowed the students to fearlessly make mistakes and then discuss them, allowing for greater understanding and a way to see their own thought processes. This also was supported by the CHPL (2005) that community classrooms where students are made to feel comfortable helped the students succeed because they were not afraid of taking risks in an effort to learn better. The students in these classrooms definitely took risks in finding ways to solve the day's problem.

The benefit of doing this intervention at an alternative site was also evident. As stated by Edwards (2013), one of the things that are important in an alternative site is a drive to continuously improve the student experience. I believe that this was shown in the intervention by the amount of students who were engaged in the lesson and willing to work on the problem at hand. Means (2015) also applauded the flexibility of alternative schools; the willingness and ability to do these talks with the full support of the administration was clear in this intervention. The flexibility the teachers at these schools have to plan and implement lessons is not necessarily reflected in the more mainstream schools. Ross (2014) also stated flexibility was key in alternative school curriculums and a focus on what the students need to earn their diplomas and learn relevant material was reflected in the intervention as well.

Looking towards the future, I believe there is ample proof for integrating more frequent number talks into alternative school curriculums. As relevance is a key for many of these students, teachers at these schools could use this time not only to drive curriculum, but also to shore up vital skills that many of these students lack, or just never learned fully earlier in their educational careers. Ensuring these students leave these schools with the skills they need to succeed is vital, not only in ensuring these students can get a job, but also to help end the cycle of poverty that can result from a lack of education.

Summation

Although this capstone did not fully answer the question, "Can frequent use of math/number talks increase the comprehension, understanding, and fluency of fractions, decimals, and percentages in alternative high school students?" it did highlight the importance of having discussions in these classes. The students were engaged during the talks and I feel the students had a better grasp of the material as a result of the talks. The data showed a modest gain in aggregate average scores between the pre- and post-test and a greater feeling of understanding from the students as reported by the surveys. The outside observer noticed that during the talks the students were engaged and willing to share their ideas with the class as a whole.

I feel this is an area that needs to be studied more. As stated before, the literature on high school number talks is almost non-existent, and even less well studied in alternative schools. I feel the results of this intervention could be supplemented by a larger and longer intervention. Once high school students, and alternative high school students, see the value of classroom discussion on various mathematical concepts they will demand more of them in their classes and help foster a more positive attitude about math. Number talks are worthwhile, even though the data of this capstone is not conclusive; there is evidence that the talks had a positive impact.

APPENDIX A

Pre- and Post-Test on fractions, decimals and percentages

Directions: Answer all questions to the best of your ability; show work when appropriate. Calculators are not allowed. If you are unsure how to do a problem, skip it.

<u>Fractions</u>

1. Simplify 6/27
2. Simplify 32/52
3. What is ¼ of 28?
4. What is 1/3 of 42?
5. 4/5+3/4=
6. 7/8-2/3=
7. 2/3×3/4=
8. 5/6÷2/3=
9. Are these two fractions, 9/12 and 24/32, equivalent? How do you know?
10. Which is bigger, 12/21 or 13/22? How do you know?

<u>Decimals</u>

11. What is .75 of 20?
12. What is .67 of 27?
13. 1.36+2.73=
14. 5.83-3.42=
15. 0.20×5=

16. 8.1÷9=
17. Round this to the nearest 100th: 2.58733
18. Round this to the nearest 10th: 2.34954
19. Round this to the nearest 1000th: 4.54274
20. What would each of three people receive if you split $480.75 evenly among them?

Percentages

21. If I had 75% of 40 apples how many apples would I have?
22. If you bought something "buy one get one half off" what is the percentage off on both items?
23. Your bill at a restaurant is $32.44, the service was decent and you want to leave a 15% tip. What would the total bill be?
24. Your bill at a restaurant is $43.78, the service was excellent and you want to leave a 20% tip. What would the total bill be?
25. What is the decimal equivalent of 28%?
26. What is the decimal equivalent of 0.1%?
27. You went to a sale and an item you wanted was 40% off. If that item originally cost $80, what is the new price?
28. You earn $10.25 an hour at your job; it was determined at your performance review that you are getting a 2% raise. What is your new hourly wage?

APPENDIX B
Qualitative PRE-Survey

Rate each statement based on the statements that follow.

1. *How do feel about working with fractions?*
    a. I actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with fractions.
    e. I love fractions and can easily work with them.
2. *How do you feel about working with decimals?*
    a. I actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with decimals.
    e. I love decimals and can easily work with them.
3. *How do you feel about percentages?*
    a. I actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with percentages.
    e. I love percentages and can easily work with them.
4. *How well do you understand the relationships between fractions, decimals and percentages?*
    f. I did not realize that there is a relationship between these concepts.
    g. I have a general idea that they are connected but have no idea how.
    h. I know how two of these concepts relate but am unsure how all three relate.
    i. I know they are connected and could figure out how to change between them if needed.
    j. I understand how these three concepts are related and use those relationships regularly.

APPENDIX C

Qualitative POST-Survey

Please answer the following questions and reflect on how frequent number talks may have changed your perceptions of the following concepts.

1. After engaging in frequent number talks how do feel about working with fractions?
    a. I still actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with fractions.
    e. I love fractions and can easily work with them.
2. After engaging in frequent number talks how do you feel about working with decimals?
    a. I still actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with decimals.
    e. I love decimals and can easily work with them.
3. After engaging in frequent number talks how do you feel about percentages?
    a. I still actively avoid and hate them.
    b. If I have to work with them I will.
    c. I am not comfortable with them but I can work with them.
    d. I have no problems working with percentages.
    e. I love percentages and can easily work with them.
4. After engaging in frequent number talks how well do you understand the relationships between fractions, decimals and percentages?
    a. I still do not understand how they relate to one another.
    b. I have a general idea that they are connected but still have no idea how.
    c. I now know how two of these concepts relate but am unsure how all three relate.
    d. I now know they are connected and could figure out how to change between them if needed.
    e. I understand how these three concepts are related and can use those relationships.

Is there anything else you wish to share about the number talks we conducted in class.

APPENDIX D

Date:
What is the specific question/task asked?

Anticipating: Likely student responses to challenging mathematical tasks

Monitoring: Students' actual responses to the tasks (while students work on the tasks in pairs or small groups):

Selecting: Particular students to present their mathematical work during the whole-class discussion

Sequencing: The student responses that will be displayed in a specific order

Connecting: Different students' responses and connecting the responses to key mathematical ideas

5 practices for orchestrating productive mathematics discussions

Date: ______________________

Task	Strategy Type		Diagrams/Visuals	Teacher Prompts/Questions
	Relational Thinking-	Procedural Explanation -		
Student				
Student				
	Strategy Type		Diagrams/Visuals	
	Relational Thinking-	Procedural Explanation -		
Student				

Student				

Depending on the question at hand there are numerous methods to get to an answer. Using number talks we can expand on the strategy offered by the student and foment greater understanding of these relationships.

Relational Thinking - What type of thinking are they showing, multiplicative or additive? Are they thinking beyond just the procedure and showing understanding of the relationships of the numbers in question? Vocabulary use is more related to conceptual understanding rather than rote procedural operations, and spatial reasoning (e.g. drawings showing how would one share two things with three people), thinking and explanations are more proportional, rather than additive, in nature.

Procedural Explanation - Focus of explanations are about rules, steps to follow, and basic use of the operations having to do with the current problem.

Diagrams/ Visuals - What artifacts were generated? How did these artifacts move the conversation among the group? These will be collected to enhance the above data.

Teacher Prompts - Space to record impromptu questions/prompts to guide student reasoning.

This field note document serves to augment items 3 and 4 on the outside observer's Look-Fors recording sheet by capturing student reasoning as evidenced through their language and visual artifacts. Additionally, this will serve as in-action field notes capturing student commentary and actions, which will be expanded after class within the context of the teacher reflection journal and triangulated to the outside, observe notes. These notes will supplement one out of the two number talks per week and will always coincide with the outside observer's notes.

APPENDIX E

March 20th, 2017

To the students and parents/guardians of the students enrolled in Mark Duffy's Statistics and Probability Course,

As your student's teacher in this course, I have undertaken a research study to determine the effectiveness of number talks in my classroom. This research is part of my classwork to complete my Masters Degree at Hamline University. Research conducted in my classroom will be included in my capstone. This research is public scholarship. The final capstone will be catalogued in Hamline's Bush Library Digital Commons, a searchable electronic repository as well as potentially presented at a conference.

My topic is on how number talks can increase students' fluency when working with fractions, decimals and percentages. Number talks are a technique to help students better understand math through structured conversations around core mathematical ideas. I am investigating how this intervention will support the students' understanding and usage of these basic concepts in a more fundamental manner thus increasing their abilities in future math courses and the workplace.

As a member of the class, I am seeking your permission to include your student's work as part of my data collection process. This will require nothing extra from your student outside of my class. I assure you that only the general results of my data collection will be included in my capstone. The identities of your student and the data collected will remain confidential. The final document will not mention the school, the district, or the state so as to maintain complete confidentiality in all respects.

The types of data collected are test score data on a pre- and post-test as well as student written and verbal reflections on fractions, decimals and percentages. Although the number talks are an integral part of my class, the use of data from your student is purely voluntary and information will not be included in the study's results unless given active consent by you. Whether or not permission is granted to use your student's information, the data collected will in no way negatively impact the instruction or final grade in my class.

Permission to conduct this research has been approved by the school district as well as Hamline University. I am required to receive your permission to use your student's data before I use the data I will collect.

If you have any questions regarding this project, please contact me directly using the information below.

With Respect,

Mark Duffy

PLEASE SIGN AND RETURN TO THE CLASSROOM

Informed Consent to Collect Classroom Data

I grant consent for my student's data and work to be used as part of Mark Duffy's Capstone through Hamline University. This data will only be used in conjunction with Mr. Duffy's study of the effectiveness of number talks and for no other reason.

____

Student Name - Printed

____

Your Name - Printed

_______________________________________ ____________________

Your Signature Date

(Researcher Copy)

Keep This Copy For Your Records

Informed Consent to Collect Classroom Data

I grant consent for my student's data and work to be used as part of Mark Duffy's Capstone through Hamline University. This data will only be used in conjunction with Mr. Duffy's study of the effectiveness of number talks and for no other reason.

____

Student Name - Printed

____

Your Name - Printed

___________________________________ __________________

Your Signature Date

(Participant Copy)

REFERENCES

Boaler, Jo and Dweck, Carol. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* San Francisco, CA: Jossey-Bass.

Committee on How People Learn (2004; 2005). *How students learn: Mathematics in the classroom.* Washington D.C.: National Academies Press.

Dickens, R. D. (2011). *Five-year trends in 10th grade alternative high school students' reading and mathematics state-mandated test scores, school adequate yearly progress, and dropout rates* (Ph.D.). Available from ProQuest Dissertations & Theses Global. (870051597). Retrieved from http://search.proquest.com/docview/870051597?accountid=28109

Dunn, T. K. (2004). Enhancing mathematics teaching for at-risk students: Influences of a teaching experience in alternative high school. *Journal of Instructional Psychology, 31*(1), 46-52. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=507892062&site=ehost-live

Edgar-Smith, S. and Palmer, R. B. (2015). Building supportive school environments for alternative education youth. *Preventing School Failure, 59*(3), 134-141. doi:10.1080/1045988X.2013.865587

Edwards, C. R. (2013). *Designing innovative alternatives to traditional high schools: What leaders need to know* (Ed.D.). Available from ProQuest Dissertations &

Theses Global. (1426639035). Retrieved from http://search.proquest.com/docview/1426639035?accountid=28109

Empson, Susan B. and Levi, Linda (2011). *Extending Children's Mathematics: Fractions and Decimals*. Portsmouth, NH: Heinemann.

Faessler, L. N. (2014). *Innovation without change or extinction: An institutional analysis of the persistence of alternative schools* (Ph.D.). Available from ProQuest Dissertations & Theses Global. (1561545268). Retrieved from http://search.proquest.com/docview/1561545268?accountid=28109

Felix, L. L. (2012). *Comparing graduation rates in alternative high schools and traditional high school* (M.A.). Available from ProQuest Dissertations & Theses Global. (1033783883). Retrieved from http://search.proquest.com/docview/1033783883?accountid=28109

Frenkel, Edward (2013). *Love and math: The heart of hidden reality.* New York, NY: Basic Books.

Glassett Farrelly, S. (2013). *Understanding alternative education: A mixed methods examination of student experiences* (Ed.D.). Available from ProQuest Dissertations & Theses Global. (1318655891). Retrieved from http://search.proquest.com/docview/1318655891?accountid=28109

Greene, D. A. (2014). *An alternative to the alternatives: Considering a progressive possibility for alternative schools* (Ph.D.). Available from ProQuest Dissertations &

Theses Global. (1553207293). Retrieved from http://search.proquest.com/docview/1553207293?accountid=28109

Harris, P. W. (2011). *Building powerful numeracy for middle and high school students*. Portsmouth, NH: Heinemann.

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. Hoboken, NJ: Routledge Ltd - M.U.A. doi:10.4324/9780203181522

Herndon, J. S. 1. and Bembenutty, H. (2014). In-school and social factors influencing learning among students enrolled in a disciplinary alternative school. *Learning & Individual Differences, 35*, 49-55. doi:10.1016/j.lindif.2014.07.007

Humphreys, Cathy. and Parker, Ruth E. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, grades 4-10.* Portland, ME: Stenhouse Publishers.

Koellner-Clark, K. and Newton, J. (2003). Flower power: Creating an engaging modeling problem to motivate mathematics students at an alternative school. *Mathematics Teacher, 96*(6), 430-433. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=507848091&site=ehost-live

Lamon, Susan J. (2008). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* New York, NY: Routledge.

Means, L. G. (2015). *The essence of alternative education: A phenomenological study of the lived experiences of at-risk secondary alternative school students* (Ph.D.). Available from ProQuest Dissertations & Theses Global. (1690277204). Retrieved from http://search.proquest.com/docview/1690277204?accountid=28109

Okamoto, Y. (2015). *The implementation process and impact of a six-week number talk intervention with sixth-grade middle school students in a large urban school district* (Ed.D.). Available from ProQuest Dissertations & Theses Global. (1691345397). Retrieved from http://search.proquest.com/docview/1691345397?accountid=28109

Parrish, S. D. (2011). Number talks build numerical reasoning. *Teaching Children Mathematics, 18*(3), 198-206. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=525502365&site=ehost-live

Parrish, S. (2010). *Number talks: Helping children build mental math and computation strategies, grades K-5*. Sausalito, CA: Math Solutions.

Ross, M. (2014). *A critical examination of the process of modifying the curriculum of alternative secondary educational programs* (Ed.D.). Available from ProQuest Dissertations & Theses Global. (1512221790). Retrieved from http://search.proquest.com/docview/1512221790?accountid=28109

Schussler, D. L. (2009). Beyond content: How teachers manage classrooms to facilitate intellectual engagement for disengaged students. *Theory into Practice, 48*(2), 114-121. doi:10.1080/00405840902776376

Schwartz, Judah L. and Kenney, Joan M. (2012). *Getting from arithmetic to algebra: Balanced assessments for the transition*. New York, NY: Teachers College Press, Columbia University.

Smith, J.P. III (2002). Development of Students' Knowledge of Fractions and Ratios. In B. Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook*. Pp3-17. Reston, VA: National Council of Teachers of Mathematics.

Susperreguy Jorquera, M. I. (2013). *"Math talk" in families of preschool-aged children: Frequency and relations to children's early math skills across time* (Ph.D.). Available from ProQuest Dissertations & Theses Global. (1497967277). Retrieved from http://search.proquest.com/docview/1497967277?accountid=28109

Washington, E. A. (2015). *The impact of the number talks strategy on student performance and attitudes of teachers and administrators in grades 3-8* (Ed.D.). Available from ProQuest Dissertations & Theses Global. (1778511147). Retrieved from http://search.proquest.com/docview/1778511147?accountid=28109

Wilkerson, K. L. 1., Afacan, K., Yan, M., Justin, W., and Datar, S. D. 1. (2016). Academic Remediation–Focused alternative schools. *Remedial & Special Education, 37*(2), 67-77. doi:10.1177/0741932515620842